

FINAL HANDOUT
MATH 349 .

1. Is the sequence $a_n = \frac{n^3}{3^n}$ convergent, ult.monotonic,bounded? Find uper and lower bounds.
2. Find the interval of convergence for $\sum_{n=1}^{\infty} \frac{(3x - 1)^n}{3^{3n}\sqrt{n}}$.
3. Find the Taylor series of $f(x) = xe^x$ around the centre $c = -1$.
For which x is the represeation valid?
4. For the curve c given as the intersection of two surfaces
 $c = \{z = x^2 + y^2\} \cap \{6x - 2y - z = 1\}$
 - (a) find a parametrization;
 - (b) find an equation of the tangent line at $P(0, -1, 1)$;
 - (c) set up the integral for evaluating arclength between the points P and $R(3, 2, 13)$. Do not evaluate.
5. Find an equation of the tangent plane to the surface
 $z = x^{y^{x^2}}$ at the point $x = 1, y = -1$.
6. For the function f defined as follows
 $f(x, y) = \frac{x^2}{x^2 + 3y^2}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$
 - (a) find the gradient ∇f at the origin;
 - (b) is f continuous at the origin? Explain.
7. Find the domain of definition,range and level curves ($c = 0, 1, -1$)
for $z = \ln \frac{x}{y}$.
8. Find the directional derivative of $f(x, y, z) = e^{-y}(x^2 + \cos z)$
at the point $A(2, 0, \pi)$ in the direction towards the point $B(-1, -1, 0)$.
9. Given $f(2, -1) = 11, f_x(2, -1) = -7, f_y(2, -1) = 5, f(2, -2) = 9,$
 $f_x(2, -2) = 3, f_y(2, -2) = -2,$ where all partials are continous, and $\mathbf{g}(u, v) = (uv^2, \frac{u}{v})$
find $\frac{\partial F}{\partial u}$ at the point $(2, -1)$ if $F = f \circ \mathbf{g}$ i.e. $F(u, v) = f(x, y)$ where $x = uv^2, y = \frac{u}{v}$.
10. The temperature T at points (x, y) of the xy - plane is given by $T(x, y) = y^2 - 3x^2$
 - (a) Draw (find) the isotherms (level curves) $T = 0, 3, -6$.

- (b) In what direction should an ant sitting at $(-1, 2)$ move to cool off as quickly as possible?
- (c) In what direction(s) is No change in the temperature?

11. Show that the systems of equations

$$xy^2 - z + u^2 = 3$$

$$x^3z + 2y - u = 0$$

$$xu + y - xyz = 3$$

can be solved for x, y, z as functions of u around the point $(x, y, z, u) = (1, 1, -1, 1)$.

12. Find the Taylor polynomial $T_2(x, y)$ for the function

$$f(x, y) = \ln(y \cos x) \text{ around the center } (0, 1).$$

13. Show that the equation $\arcsin(zy) + z^3x + x^2y + 8 = 0$

can be solved for z as a function of x, y around the point $P(-1, 0, 2)$

then find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ at that point..