

MATH 349
Handout #4

1. For $f(x, y) = \sqrt{2x + y^2}$ find the range; sketch the domain;
sketch the level curves for $c = 0, -1, 2$;
and show that $y f_x = f_y$ in the domain (f_x, f_y are partial derivatives).
2. Find $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3}{x^2 + 2y^2}$, if it exists.
3. For $f(x, y) = \frac{e^y}{x}$
 - (a) sketch the domain and level curves for $c = 0, \pm 1, e$ in the xy -plane
 - (b) find the second mixed partial derivative f_{xy} .
4. Show that $f(x, y) = \frac{1}{\sqrt{yx^2 + y^2 + \frac{1}{4}x^4}}$ satisfies the equation
 $f_x = x f_y$ in the domain - find it!
5. For $f(x, y) = \ln(x^2 + y^2 + x)$ find the range; sketch the domain of f ;
and sketch the level curves of f for $c = 0, 1, \ln 2, -\ln 2, \dots$
6. Find $\lim_{(x,y) \rightarrow (-1,0)} \frac{xy + y}{(x+1)^2 + y^2}$ as $(x, y) \rightarrow (-1, 0)$ if it exists.
7. For $f(x, y) = \arctan \frac{x}{y}$ show that $y \cdot f_x - x \cdot f_y = 1$ for any x and $y \neq 0$.
(f_x and f_y denote partial derivatives with respect to x and y respectively)
8. For $f(x, y) = \frac{2x}{x^2 + y}$ sketch the domain of f ;
sketch in the xy -plane the level curves for $c = 0, -2, 1$;
and find an equation of the tangent plane to $z = f(x, y)$ at $x = -1, y = 1$.
9. Find $\lim_{(x,y) \rightarrow (0,1)} \frac{xy - x}{3x^2 + 2(y-1)^4}$, if it exists.