

MATH 349
Midterm Handout

1. Determine if the indicated sequence is bounded, ult.monotonic, and convergent

a) $a_n = \frac{\ln(n+3)}{n+3}$ b) $b_n = \frac{n^n}{n!}$.

2. Determine whether the indicated series is absolutely convergent,

conditionally convergent or divergent. (a) $\sum_{k=1}^{\infty} \frac{\arctan k}{1+k^2}$; (b) $\sum_{k=1}^{\infty} (-1)^k \frac{\arctan k}{k}$

(c) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$ (d) $\sum_{k=1}^{\infty} k^2 e^{-k}$ (e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{(n^2 - 2n)\sqrt{n}}$

3. Find the interval of convergence if

(a) $\sum_{k=1}^{\infty} \frac{2^k}{\sqrt{k}} (x-1)^k$ b) $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$ (only abs.convergence).

4. Find the sum of (a) $\sum_{k=1}^{\infty} \frac{(\ln 2)^k}{k!}$ (b) $\sum_{n=3}^{\infty} \frac{(-1)^n}{2^n(n+1)}$.

(a) Is the sequence $a_n = \frac{2 - (-1)^n}{n^2 - 2n}$, $n \geq 3$ bounded, alternating or convergent?

(b) Is the sequence $c_n = \frac{3^n}{3^n - 2^n}$ convergent? Is the series $\sum_{n=1}^{\infty} c_n$ convergent?

5. Find the Taylor series for $f(x) = \frac{1}{(x+3)x}$ around the center $x_0 = -1$, particularly the coefficient a_6 .

For what values of x is the representation valid? (Hint: Use partial fractions)

6. Find Taylor polynomial of degree 3 for $f(x) = \ln \frac{x-1}{x}$ around the centre $x_0 = 2$.

7. Find a parametrization of the curve c given as the intersection of the cone $\{z = \sqrt{2x^2 + 2y^2}\}$ and the plane $\{z + x = 1\}$.

8. For the curve c given by $\mathbf{r}(t) = (2t, t^2, \ln t)$, $t > 0$ find

(a) an equation of the tangent line at $P(2, 1, 0)$;

(b) the arclength of c between P and $R(2e, e^2, 1)$.

9. For the curve c given by $\mathbf{r}(t) = (t \sin t, t \cos t, 2t)$

(a) find an equation of the tangent line to c at the origin;

(b) find the arclength between the origin and the point $A\left(\frac{\pi}{2}, 0, \pi\right)$.

10. Find a parametrization of the curve c given as the intersection of two surfaces

$c = \{x^2 + y^2 = 2z\} \cap \{3x - 4y - z = 0\}$.