FINAL HANDOUT
MATH 349 .

1. Is the sequence $a_n = \dfrac{n^3}{3^n}$ convergent, ult.monotonic,bounded? Find uper and lower bounds.

2. Find the interval of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{(3x-1)^n}{3^{3n}\sqrt{n}}$.

3. Find the Taylor series of $f(x) = xe^x$ around the centre $c = -1$.
   For which x is the represenation valid?

4. For the curve c given as the intersection of two surfaces
   $c = \{z = x^2 + y^2\} \cap \{6x - 2y - z = 1\}$

   (a) find a parametrization;

   (b) find an equation of the tangent line at $P\,(0, -1, 1)$;

   (c) set up the integral for evaluating arclength between the points P and $R\,(3, 2, 13)$ .Do not evaluate.

5. Find an equation of the tangent plane to the surface
   $z = x^{yx^2}$ at the point $x = 1, y = -1$.

6. For the function f defined as follows
   $f(x, y) = \dfrac{x^2}{x^2 + 3y^2}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$

   (a) find the gradient ∇f at the origin;

   (b) is f continuous at the origin? Explain.

7. Find the domain of definition,range and level curves ($c = 0, 1, -1$)
   for $z = \ln \frac{x}{y}$.

8. Find the directional derivative of $f\,(x, y, z) = e^{-y}\,(x^2 + \cos z)$
   at the point $A\,(2, 0, \pi)$ in the direction towards the point $B\,(-1, -1, 0)$.

9. Given $f\,(2, -1) = 11$, $f_x\ (2, -1) = -7$,$f_y\,(2, -1) = 5$, $f(2, -2) = 9$,
   $f_x(2, -2) = 3$, $f_y(2, -2) = -2$,where all partials are continous, and $\mathbf{g}\,(u, v) = \left(uv^2, \frac{u}{v}\right)$
   find $\dfrac{\partial F}{\partial u}$ at the point $(2, -1)$ if $F = f \circ \mathbf{g}$ i.e. $F(u, v) = f(x, y)$ where $x = uv^2, y = \frac{u}{v}$.

10. The temperature T at points (x, y) of the $xy-$ plane is given by $T\,(x, y) = y^2 - 3x^2$

    (a) Draw (find) the isotherms (level curves) $T = 0, 3, -6$.

(b) In what direction should an ant siting at $(-1, 2)$ move to cool off as quickly as possible?

(c) In what direction(s) is No change in the temperature?

11. Show that the systems of equations

$$xy^2 - z + u^2 = 3$$
$$x^3 z + 2y - u = 0$$
$$xu + y - xyz = 3$$

can be solved for x, y, z as functions of u around the point $(x, y, z, u) = (1, 1, -1, 1)$.

12. Find the Taylor polynomial $T_2(x, y)$ for the function $f(x, y) = \ln(y \cos x)$ around the center $(0, 1)$.

13. Show that the equation $\arcsin(zy) + z^3 x + x^2 y + 8 = 0$ can be solved for z as a function of x, y around he point $P(-1, 0, 2)$ then find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ at that point..