The University of Calgary
Department of Mathematics and Statistics
MATH 349 Lec 01/02
Quiz # 1T

Fall 2007

Name:____________________________**I.D.#:**__________________

JUSTIFY YOUR ANSWERS.
Answer each question in the space provided.
A correct answer without work shown may be worth 0 points,
while an incorrect answer with full justifiction may be worth partial credit.

1. Let $a_n = \dfrac{n}{\ln n}$ for $n \geq 2$.

   Is the sequence ultimately monotonic, bounded and convergent?Explain. [5]

   for $n \geq 2$ $\quad \ln n > 0 \quad \rightarrow a_n > 0 \quad \lim\limits_{n\to\infty} \dfrac{n}{\ln n} = (\text{L'H.R.}) \lim\limits_{x\to\infty} \dfrac{1}{x^{-1}} = \lim\limits_{x\to\infty} x = +\infty$

   the sequence is **divergent**, **not bounded above, bounded below by 0**

   for monotonicity

   for $x \geq 2 \quad f(x) = \dfrac{x}{\ln x} \quad f'(x) = \left(\dfrac{x}{\ln x}\right)' = \dfrac{\ln x - 1}{(\ln x)^2} > 0 \quad$ if $\ln x > 1, x > e$

   thus the sequence is increasing for $n \geq 3$ **ult.incr.**

2. Let $b_n = \dfrac{n}{n^2+36}$ for $n \geq 1$. Is the sequence $\{b_n\}$ultimately monotonic, bounded, alternating, convergent? [5]

   $b_n = \dfrac{n}{n^2+36} > 0 \quad \lim\limits_{n\to\infty} \dfrac{n}{n^2+36} = \lim\limits_{n\to\infty} \dfrac{\frac{1}{n}}{1+\frac{36}{n^2}} = 0$ or L'H.R., also we can see that $\dfrac{n}{n^2+36} < 1$

   the sequence is **convergent and bounded**

   define $\quad f(x) = \dfrac{x}{x^2+36} \quad f'(x) = \left(\dfrac{x}{x^2+36}\right)' = \dfrac{x^2+36-2x^2}{(x^2+36)^2} = \dfrac{36-x^2}{(x^2+36)^2} < 0$ for $x > 6$

   so the sequence is decreasing for $n > 6$ **ult.decr.**

   also we can prove $\quad b_{n+1} < b_n \quad \dfrac{n+1}{n^2+2n+37} < \dfrac{n}{n^2+36} \quad (n+1)\left(n^2+36\right) < n\left(n^2+2n+37\right)$

   $n^3+n^2+36n+36 < n^3+2n^2+37n \quad 36 < n^2+n = n(n+1)$ true for $n \geq 6$

3. Evaluate the limit $\lim_{n\to\infty} \frac{4^n}{n^n}$. Is the sequence bounded?

If so,provide a lower and an upper bound. [5]

$\lim_{n\to\infty} \frac{4^n}{n^n} = \lim_{x\to\infty} \frac{4^x}{x^x} = \lim_{x\to\infty} \frac{e^{x \ln 4}}{e^{x \ln x}} = \lim_{x\to\infty} e^{x(\ln 4 - \ln x)} = "e^{-\infty}" = 0$ since for $x > 4$ the exponent is negative

by Theorem the sequence is bounded, (also decr. for $n \geq 5$)

for $n \geq 5$ $\quad 0 < \frac{4^n}{n^n} \leq \left(\frac{4}{5}\right)^n < 1 \qquad b_n = \left(\frac{4}{5}\right)^n$ is a geom,sequ. convergent to 0, so by Squ.Th

also the original sequ. is convergent to 0; for bounds:

$a_1 = 4, a_2 = 4, a_3 = \frac{4^3}{3^3} = \frac{64}{27} < 4 \qquad a_4 = 1 \qquad a_5 < 1 \rightarrow \qquad 0 < a_n \leq 4$