

The University of Calgary
Department of Mathematics and Statistics
MATH 349 -01/02
Quiz # 2T

Fall, 2007

Name: _____ I.D.#: _____

1. Find the sum of the series $\sum_{n=2}^{\infty} 2^n 3^{1-n} = 3 \sum_{n=2}^{\infty} \frac{2^n}{3^n} = 3 \cdot \frac{\left(\frac{2}{3}\right)^2}{1 - \frac{2}{3}} = 4$ using $\sum_{n=N}^{\infty} r^n = \frac{r^N}{1-r}$ for $-1 < r < 1$.

2. Is the series $\sum_{n=2}^{\infty} \frac{3^n \ln n}{n!}$ convergent or divergent? Explain.

By Ratio Test: $\frac{a_{n+1}}{a_n} = \frac{3^{n+1} \ln(n+1)}{(n+1)n!} \cdot \frac{n!}{3^n \ln n} = \frac{3}{n+1} \cdot \frac{\ln(n+1)}{\ln n} \rightarrow 0 \cdot 1 = 0$ since

$\lim_{x \rightarrow \infty} \frac{\ln(x+1)}{\ln x} = 1$ by L'H.R.

the limit is less than 1 so convergent.

3. Is the series $\sum_{n=2}^{\infty} \ln\left(\frac{n}{2n+1}\right)$ convergent or divergent? Explain.

4. Since $\lim_{n \rightarrow \infty} \ln\left(\frac{n}{2n+1}\right) = \ln\left(\lim_{n \rightarrow \infty} \left(\frac{n}{2n+1}\right)\right) = \ln \frac{1}{2} \neq 0$ $\lim_{n \rightarrow \infty} \frac{n}{2n+1} = \lim_{n \rightarrow \infty} \frac{1}{2 + \frac{1}{n}} = \frac{1}{2}$

the series is divergent.