MATH 349
Handout #4

1. For $f(x,y) = \sqrt{2x+y^2}$ find the range; sketch the domain; sketch the level curves for $c = 0, -1, 2$; and show that $y\ f_x = f_y$ in the domain (f_x, f_y are partial derivatives).

2. Find $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3}{x^2+2y^2}$,if it exists.

3. For $f(x,y) = \dfrac{e^y}{x}$

   (a) sketch the domain and level curves for $c = 0, \pm 1, e$ in the xy-plane

   (b) find the second mixed partial derivative f_{xy}.

4. Show that $f(x,y) = \dfrac{1}{\sqrt{yx^2+y^2+\frac{1}{4}x^4}}$ satisfies the equation $f_x = x f_y$ in the domain - find it!

5. For $f(x,y) = \ln(x^2+y^2+x)$ find the range; sketch the domain of f; and sketch the level curves of f for $c = 0, 1, \ln 2, -\ln 2, \ldots$.

6. Find $\lim \dfrac{xy+y}{(x+1)^2+y^2}$ as $(x,y) \to (-1,0)$ if it exists.

7. For $f(x,y) = \arctan \frac{x}{y}$ show that $y \cdot f_x - x \cdot f_y = 1$ for any x and $y \neq 0$. (f_x and f_y denote partial derivatives with respect to x and y respectively)

8. For $f(x,y) = \dfrac{2x}{x^2+y}$ sketch the domain of f; sketch tn the xy-plane the level curves for $c = 0, -2, 1$; and find an equation of the tangent plane to $z = f(x,y)$ at $x = -1, y = 1$.

9. Find $\lim\limits_{(x,y)\to(0,1)} \dfrac{xy-x}{3x^2+2(y-1)^4}$,if it exists.