The University of Calgary
Department of Mathematics and Statistics
MATH 349 Handout # 5

1. For $f(x,y) = \dfrac{xy}{\sqrt{1+x^2}}$ and $\mathbf{g} : R^2 \to R^2, \mathbf{g}(s,t) = (\cos(\pi st), \sin\dfrac{\pi s}{t})$

   (a) find ∇f;

   (b) using Chain Rule find $\nabla h(0,-1)$ where $h = f \circ \mathbf{g}$
   (or $h(s,t) = f(x,y)$ where $x = \cos(\pi st)$ and $y = \sin\dfrac{\pi s}{t}$)

2. Find an equation of the tangent plane to $z = f(x,y) = \ln(x+y^2)$ at the point $x_0 = 0, y_0 = -1$.

3. For the function $f(x,y) = e^{\sqrt{\frac{y}{x}}}$ find the domain and f_{xx} and f_{xy}.

4. For $f(x,y,z) = \sqrt{2}\sin(\pi xy + x\ln z)$ and $\mathbf{g} : R \to R^3, \mathbf{g}(t) = (\frac{1}{t}, -\frac{1}{t}, \frac{t}{2})$

   (a) find ∇f;

   (b) find $D\mathbf{g}$ or $\mathbf{g}'$

   (c) using Chain Rule find $h'(2)$ where $h = f \circ \mathbf{g}$
   (or $h(t) = f(x,y,z)$ where $x = \frac{1}{t}$, $y = \frac{-1}{t}$ and $z = \frac{t}{2}$)

5. In what directions at the point $P(2,1)$ does the function $f(x,y) = \ln\left(\frac{x}{y} + \frac{y}{x}\right)$ have the rate of change equal to $\frac{3}{10}$?What is the maximum rate of change at that point?

6. Find the rate and the direction of the most rapid decrease of $f(x,y,z) = x^2ze^y + xz^2$ at the point$P(1, \ln 2, 2)$.

7. Given $f(x,y) = \begin{cases} \dfrac{xy}{2x^2+y^2} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{at } (0,0) \end{cases}$

   (a) Is f continous at $(0,0)$?

   (b) Find ∇f at $(0,0)$, if it exists.

   (c) Find the directional derivative at $(0,0)$ in the direction of $y = x$ if it exists.

   (d) Find the directional derivative at $(-1,-1)$ where $\mathbf{u} = \dfrac{1}{\sqrt{2}}(1,1)$.