The University of Calgary
Department of Mathematics and Statistics
MATH 349 Lec 01/02
Quiz # 1R

Fall 2007

Name:____________________________**I.D.#:**__________________

JUSTIFY YOUR ANSWERS.
Answer each question in the space provided.
A correct answer without work shown may be worth 0 points,
while an incorrect answer with full justifiction may be worth partial credit.

1. Let $a_n = \dfrac{n^2}{e^n}$ for $n \geq 1$.

   Is the sequence ultimately monotonic, bounded and convergent?Explain. [5]

   $a_n > 0 \qquad \lim\limits_{n\to\infty} \dfrac{n^2}{e^n} = \text{(L'H.R.twice)} \lim\limits_{x\to\infty} \dfrac{2x}{e^x} = \lim\limits_{x\to\infty} \dfrac{2}{e^x} = 0$

   the sequence is **convergent**,therefore **bounded above and bounded below (**by 0**)** for monotonicity

   for $x \geq 1 \qquad f(x) = \dfrac{x^2}{e^x} = x^2e^{-x} \qquad f'(x) = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2-x) < 0$ if $x > 2$

   thus the sequence is decreasing for $n \geq 3$ **ult.decr.**

2. Let $b_n = \dfrac{\cos n}{n^2}$ for $n \geq 1$. Is the sequence $\{b_n\}$ultimately monotonic, bounded, alternating, convergent? [5]

   b_n is sometimes positive,sometimes negative so it is **NOT monotonic, neither alternating**: $b_1 > 0, b_2 < 0, b_3 < 0,$

   $b_4 < 0, b_5 > 0, b_6 > 0, b_7 > 0, 3neg, 3pos, ...$

   $\lim\limits_{n\to\infty} \dfrac{\cos n}{n^2} = 0$ by Squ, th.,since $\qquad \dfrac{-1}{n^2} \leq \dfrac{\cos n}{n^2} \leq \dfrac{1}{n^2}$ and $\lim\limits_{n\to\infty} \dfrac{\pm 1}{n^2} = 0$

   the sequence is **convergent and bounded**

   $-1 \leq \dfrac{-1}{n^2} \leq \dfrac{\cos n}{n^2} \leq \dfrac{1}{n^2} \leq 1$

3. Evaluate the limit $\qquad \lim\limits_{n\to\infty} \left(\sqrt{2n^2+1} - n\right) \qquad$ Is the sequence bounded? [5]

   $$\lim_{n\to\infty} \left(\sqrt{2n^2+1} - n\right) = \lim_{n\to\infty} \left(\sqrt{2n^2+1} - n\right) \cdot \frac{\sqrt{2n^2+1}+n}{\sqrt{2n^2+1}+n} = \lim_{n\to\infty} \frac{2n^2+1-n^2}{\sqrt{2n^2+1}+n} =$$

   $$= \lim_{n\to\infty} \frac{n^2+1}{\sqrt{2n^2+1}+n} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n+\frac{1}{n}}{\sqrt{2+\frac{1}{n^2}}+1} = +\infty$$

   the sequence is **divergent, not bounded above , but bounded below,** $b_n > 0$.