**Department of Mathematics and Statistics**
**MATH 349-01/02**
**Quiz # 3R**

**Fall 2007**

**Name:**____________________________**I.D.#:**__________________

**Note:**Use of scientific, non-programmable calculators is allowed ;

the use of one page of notes is allowed.The quiz is 45 minutes

1. Is the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$absolutely or conditionally convergent or divergent? Explain. [5]

2. Find the centre,radius and interval of convergence of power series $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{\sqrt{n}3^n}$. [5]

3. Express $f(x) = \ln x$ in powers of $x-2$ and find the interval where the representation is valid. [5]

**Solution**

**For 1)**

first abs. convergence $\sum_{n=1}^{\infty} \left| \frac{(-1)^n n}{n^2+1} \right| = \sum_{n=1}^{\infty} \frac{n}{n^2+1}$

the sequence $a_n = \frac{n}{n^2+1} \sim b_n = \frac{1}{n}$ since

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{n^2+1} \cdot n = \lim_{n\to\infty} \frac{n^2}{n^2+1} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n^2}} = 1 \neq 0$$

and the series $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$ (harmonic series) ,therefore $\sum_{n=1}^{\infty} \frac{n}{n^2+1} = \infty$ by Limit Comp.Test

but the serie$\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$ is **cond.convergent** by Alt.Test

since $a_n > 0$, $\lim_{n\to\infty} \frac{n}{n^2+1} = 0$ (L'H.R.)

we have to show that $\{a_n\}$ is decreasing :

$$f'(x) = \left(\frac{x}{x^2+1}\right)' = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} < 0 \text{ for } x > 1$$

**For 2)**

$$\sum_{n=1}^{\infty}\frac{(3x-2)^n}{\sqrt{n}3^n}=\sum_{n=1}^{\infty}\frac{3^n(x-\frac{2}{3})^n}{\sqrt{n}3^n}=\sum_{n=1}^{\infty}\frac{(x-\frac{2}{3})^n}{\sqrt{n}}$$

The centre is $c=\frac{2}{3}$ and $a_n=\dfrac{1}{\sqrt{n}}$.Since $\left|\dfrac{a_{n+1}}{a_n}\right|=\sqrt{\dfrac{n}{n+1}}\to 1$

$R=1$ and the series is abs.convergent on the interval is $\left(-\frac{1}{3},\frac{5}{3}\right)$

Also by Root Test $(|a_n|)^{\frac{1}{n}}=\sqrt{\dfrac{1}{\sqrt[n]{n}}}\to 1$

now ,the ends at $x=\frac{5}{3}$ $\quad\displaystyle\sum_{n=1}^{\infty}\frac{(x-\frac{2}{3})^n}{\sqrt{n}}=\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}=\infty, p=\frac{1}{2}<1$

and at $x=-\frac{1}{3}$ $\quad\displaystyle\sum_{n=1}^{\infty}\frac{(x-\frac{2}{3})^n}{\sqrt{n}}=\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ **cond.convergent,** since $\dfrac{1}{\sqrt{n}}\searrow 0$

together the interval of convergence is $\left[-\frac{1}{3},\frac{5}{3}\right)$.

**For 3)**

The answer must be in the form $\displaystyle\sum_{n=0}^{\infty}a_n\,(x-2)^n$ .Rewrite $\ln(x)=\ln(2+(x-2))=$

or $\quad x-2=t \quad x=t+2=2\left(1+\frac{t}{2}\right)$

$$=\ln 2(1+\frac{x-2}{2})=\ln 2+\ln(1+\frac{x-2}{2})=\ln 2+\sum_{n=1}^{\infty}\frac{(-1)^n}{n2^n}\,(x-2)^n$$

check $\quad a_0=f(2)=\ln 2$

for $\quad -1<\dfrac{x-2}{2}\le 1$ so $\quad -2<x-2\le 2 \quad 0<x\le 4$

using $\quad\displaystyle\ln(1+s)=\sum_{n=1}^{\infty}(-1)^n\,\frac{s^n}{n}$ for $-1<s\le 1$.

also the function is defined only for $x>0$ so the radius $R=2$ distance between the centre 2 and 0.