The University of Calgary
Department of Mathematics and Statistics
MATH 349-01/02 Quiz # 3T

Fall 2007

Name:____________________________**I.D.#:**__________________

Note: Use of scientific, non-programmable calculators is allowed ; the use of one page of notes is allowed.The quiz is 45 minutes

1. Is the series $\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{n+1}$ absolutely or conditionally convergent or divergent?Explain. [5]

2. Find the centre,radius and interval of convergence of power series $\sum_{n=1}^{\infty}\frac{(3x+2)^n}{n\sqrt{n}}$ [5]

3. Express $f(x)=\frac{1}{x^2}$ in powers of $(x-3)$ On what interval is the representation valid? [5]

Solutions

For1)

First $\sum_{n=1}^{\infty}\left|(-1)^n\frac{\sqrt{n}}{n+1}\right|=\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n+1}=\infty$ since it is equivalent to $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$

to show that $a_n=\frac{\sqrt{n}}{n+1}\backsim b_n=\frac{1}{\sqrt{n}}$ $\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\frac{\sqrt{n}}{n+1}\cdot\sqrt{n}=\lim_{n\to\infty}\frac{n}{n+1}=1\neq 0$

(Limit Comp.Test)

$\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{n+1}$ is **conditionally convergent** by Alternating Test::

the sequence a_n is decreasing and limit is 0.

$\lim_{n\to\infty}\frac{\sqrt{n}}{n+1}=\lim_{n\to\infty}\frac{\frac{1}{\sqrt{n}}}{1+\frac{1}{n}}=0$ and

$f'(x)=\left(\frac{\sqrt{x}}{x+1}\right)'=\frac{\frac{1}{2\sqrt{x}}(x+1)-\sqrt{x}}{(x+1)^2}=\frac{x+1-2x}{2\sqrt{x}(x+1)^2}=\frac{1-x}{2\sqrt{x}(x+1)^2}<0$ for $x>1$

OR

$a_{n+1}<a_n$ $\frac{\sqrt{n+1}}{n+2}<\frac{\sqrt{n}}{n+1}$ $(n+1)\sqrt{n+1}<(n+2)\sqrt{n}$

square both sides

$(n+1)^3<n(n+2)^2$ $n^3+3n^2+3n+1<n^3+4n^2+4n$ $1<n^2+n$

For 2)

the centre is $c=-\frac{2}{3}$ and $a_n=\frac{3^n}{n^{\frac{3}{2}}}$ since $\sum_{n=1}^{\infty}\frac{(3x+2)^n}{n\sqrt{n}}=\sum_{n=1}^{\infty}\frac{3^n}{n^{\frac{3}{2}}}\left(x+\frac{2}{3}\right)^n$

for the radius

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{3^{n+1}}{(n+1)^{\frac{3}{2}}} \cdot \frac{n^{\frac{3}{2}}}{3^n} = 3\left(\frac{n}{n+1}\right)^{\frac{3}{2}} \to 3 \text{ as } x \to \infty, \text{ so } R = \tfrac{1}{3}$$

and the series is absolutely convergent for $x \in \left(-1, -\frac{1}{3}\right)$

now for $x = -1$ and $x = -\frac{1}{3}$ both series are also abs.convergent since $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ is convergent

p-series with $p = \frac{3}{2} > 1$.Together the series is **abs.convergent for** $x \in \left[-1, -\frac{1}{3}\right]$,otherwise divergent.

For 3)

first $\quad x - 3 = t \quad x = t + 3$

$$\frac{1}{x} = \frac{1}{t+3} = \frac{1}{3} \cdot \frac{1}{1+\frac{t}{3}} = \frac{1}{3}\sum_{n=0}^{\infty}(-1)^n\left(\frac{x-3}{3}\right)^n = \sum_{n=0}^{\infty}(-1)^n\frac{(x-3)^n}{3^{n+1}}$$

for $-1 < \frac{x-3}{3} < 1$, so $-3 < x - 3 < 3$, and finally $0 < x < 6$

(using $\sum_{n=0}^{\infty}(-1)^n r^n = \frac{1}{1+r}$ for $-1 < r < 1$) now, differentiate

$$\frac{-1}{x^2} = \sum_{n=1}^{\infty}(-1)^n\frac{n(x-3)^{n-1}}{3^{n+1}} \qquad \text{so for } 0 < x < 6$$

$$\frac{1}{x^2} = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{n(x-3)^{n-1}}{3^{n+1}} = \sum_{k=0}^{\infty}(-1)^k\frac{(k+1)(x-3)^k}{3^{k+2}}$$

$k = n - 1$