The University of Calgary
Department of Mathematics and Statistics
MATH 349-01/02 Quiz # 5R Fall 2007

Name:____________________________**I.D.#:**__________________

1. For $f(x,y) = \dfrac{x}{x^2+y^2}, f : R^2 \to R$ and $\mathbf{g}(u,v) = (\dfrac{u}{v^2}, u^2v), \mathbf{g} : R^2 \to R^2$

   find $\nabla h = \left(\dfrac{\partial h}{\partial u}, \dfrac{\partial h}{\partial v}\right)$ where $h = f \circ \mathbf{g}$ at the point $u = 2, v = -1$

   using the chain rule.

   i.e. $h(u,v) = f(x,y)$ with $x = \dfrac{u}{v^2}$ and $y = u^2 v$. [5]

2. For $f(x,y) = x^2 \ln(y^2 + e^x) + \sin(xy^2)$ find the directional derivative at $A\,(0,-1)$ in the direction from from the point $A\,(0,-1)$ to the point $B\,(1,1)$. [5]

3. Find an equation of the tangent plane to $f(x,y) = e^{y\sqrt{2x}}$ at $x_0 = 2 \quad y_0 = 0$ [5]

Solution
For 1)

first we need $\nabla f = (f_x, f_v) = \left(\dfrac{y^2 - x^2}{(x^2+y^2)^2}, \dfrac{-2xy}{(x^2+y^2)^2}\right)$ for any $(x,y) \neq (0,0)$

for $u = 2, v = -1 \qquad x = 2 \qquad y = -4$

thus

$$\nabla f\,(2,-4) = \left(\frac{12}{400}, \frac{16}{400}\right) = \left(\tfrac{3}{100}, \tfrac{4}{100}\right) = \tfrac{1}{100}\,(3,4)$$

and

$\dfrac{\partial x}{\partial u} = \dfrac{1}{v^2} = 1 \qquad \dfrac{\partial x}{\partial v} = \dfrac{-2u}{v^3} = 4 \qquad \dfrac{\partial y}{\partial u} = 2uv = -4 \qquad \dfrac{\partial y}{\partial v} = u^2 = 4$ at $u = 2, v = -1$

so

$\dfrac{\partial h}{\partial u}(2,-1) = \dfrac{\partial f}{\partial x}(2,-4)\cdot\dfrac{\partial x}{\partial u}(2,-1) + \dfrac{\partial f}{\partial y}(2,-4)\cdot\dfrac{\partial y}{\partial u}(2,-1) = \tfrac{1}{100}(3,4)\bullet(1,-4) = \dfrac{-13}{100}$

$\dfrac{\partial h}{\partial v}(2,-1) = \dfrac{\partial f}{\partial x}(2,-4)\cdot\dfrac{\partial x}{\partial v}(2,-1) + \dfrac{\partial f}{\partial y}(2,-4)\cdot\dfrac{\partial y}{\partial v}(2,-1) = \tfrac{1}{100}(3,4)\bullet(4,4) = \tfrac{280}{100} = \dfrac{7}{25}$

OR

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at $u = 2, v = -1$ we get $D\mathbf{g}\,(2,-1) = \begin{bmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ -4 & 4 \end{bmatrix}$

then $\nabla h\,(2,-1) = \nabla f\,(-2,-2)\,D\mathbf{g}\,(2,-1) = \begin{bmatrix} \dfrac{3}{100} & \dfrac{4}{100} \end{bmatrix} \begin{bmatrix} 1 & 4 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} \dfrac{-13}{100} & \dfrac{7}{25} \end{bmatrix}$

For 2)

the direction $\overrightarrow{AB} = (1,2)$ $\mathbf{u} = \frac{1}{\sqrt{5}}(1,2)$

$f_x = 2x\ln(y^2+e^x) + \dfrac{x^2e^z}{y^2+e^x} + y^2\cos(xy^2)$ $f_y = \dfrac{2x^2y}{y^2+e^x} + 2xy\cos(xy^2)$

$\nabla f\,(0,-1) = (1,0)$ both partials are cont.so

by Theorem $D_{\mathbf{u}}f(0,-1) = \nabla f\,(0,-1) \bullet \mathbf{u} = (1,0) \circ \frac{1}{\sqrt{5}}(1,2) = \dfrac{1}{\sqrt{5}}$

For 3)

for $f(x,y) = e^{y\sqrt{2x}}$ $f(2,0) = 1$ so the point is $P\,(2,0,1)$ and

$f_x = e^{y\sqrt{2x}} \cdot \dfrac{y}{\sqrt{2x}}$ since$\left[(2x)^{\frac{1}{2}}\right]' = \frac{1}{2}\,(2x)^{-\frac{1}{2}} \cdot 2$

$f_y = e^{y\sqrt{2x}} \cdot \sqrt{2x}$ $\nabla f(2,0) = (0,2)$

so a normal is $\mathbf{n} = (\nabla f(2,0)\,,-1) = (0,2,-1)$ and tangent plane $2y - z = d$

using P $0 - 1 = d$ $2y - z + 1 = 0$