The University of Calgary
Department of Mathematics and Statistics
MATH 349-01/02 Quiz # 5T Fall,2007

Name:____________________________**I.D.#:**________________

1. For $f(x,y,z) = \left(\dfrac{\ln y}{x\sqrt{z}} + \cos(\dfrac{\pi z}{2})\right)$ and $\mathbf{g}: R \to R^3, \mathbf{g}(t) = (t^2, \sqrt{t}, \frac{2}{\sqrt{t}})$
   find $h'(1)$ where $h = f \circ \mathbf{g}$ **using Chain Rule**
   HINT: $h(t) = f(x,y,z)$ where $x = t^2$, $y = \sqrt{t}$ and $z = \frac{2}{\sqrt{t}}$. [5]

2. Find an equation of the tangent plane to $f(x,y) = \ln\left(\frac{\sqrt{x^2+4y}}{3}\right)$
   at $x_0 = -1$ $y_0 = 2$ HINT: Simplify first! [5]

3. For the function. $f(x,y) = \dfrac{y}{x^4+y^2}$ find the directional derivative at $A(-1,1)$
   in the direction of the line $y = 3x+4$ (from left to right). [3]

Solutions

For 1) $\nabla f = (f_x, f_y, f_z)$

$f_x = -\dfrac{\ln y}{x^2\sqrt{z}}$, $f_y = \dfrac{1}{xy\sqrt{z}}$ $f_z = \left[-\dfrac{\ln y}{2xz\sqrt{z}} + \dfrac{\pi}{2}\sin\left(\frac{\pi}{2}z\right)\right]$ are cont.

for any $x, y > 0$ and $z > 0$.

$\mathbf{g}' = (x', y', z') = \left(2t, \dfrac{1}{2\sqrt{t}}, -\dfrac{1}{t\sqrt{t}}\right)$ at $t = 1$

$\mathbf{g}'(1) = (x'(1), y'(1), z'(1)) = \left(2, \frac{1}{2}, -1\right)$

if $t = 1$ then $x = 1, y = 1, z = 2$ and

$\nabla f(1,1,2) = \left(0, \frac{1}{\sqrt{2}}, 0\right)$ since $\sin\pi = 0$

by Chain Rule $h' = \nabla f \bullet \mathbf{g}' = 1\dfrac{1}{2\sqrt{2}}$ OR

$h'(1) = f_x(1,1,2)\,x'(1) + f_y(1,1,2)\,y'(1) + f_z(1,1,2)\,z'(1) =$

$$= 0 + \tfrac{1}{\sqrt{2}}\left(\tfrac{1}{2}\right) + 0 = \frac{1}{2\sqrt{2}.}$$

For 2)

first $f(-1,2) = \ln\left(\frac{\sqrt{x^2+4y}}{3}\right) = \ln 1 = 0$ so the point on the graph is $P(-1,2,0)$

you can simplify $f(x,y) = \ln\left(\frac{\sqrt{x^2+4y}}{3}\right) = \frac{1}{2}\ln(x^2+4y) - \ln 3$

partials $f_x = \dfrac{x}{x^2+4y}$ $f_y = \dfrac{2}{x^2+4y}$

and $\quad f_x(-1,2) = -\frac{1}{9} \qquad f_y(-1,2) = \frac{2}{9}$

and a normal is $\mathbf{n} = (a,b,c) = (\nabla f, -1) = \left(-\frac{1}{9}, \frac{2}{9}, -1\right)$or $(1,-2,9)$

$x - 2y + 9z = d$ and it goes through $P \qquad -1-4 = d$ so $d = -5$

and $\quad x - 2y + 9z + 5 = 0 \qquad$ OR

$z = f(-1,2) + \ f_x(-1,2)\,(x+1) + f_y(-1,2)\,(y-2) = \frac{-1}{9}\,(x+1) + \frac{2}{9}\,(y-2)$

For 3)

$f(x,y) = \dfrac{y}{x^4+y^2} \qquad$ *i*s defined if $(x,y) \neq (0,0)$

$f_x = \dfrac{-4x^3y}{(x^4+y^2)^2} \qquad f_y = \dfrac{1}{x^4+y^2} - \dfrac{2y^2}{(x^4+y^2)^2}$ both are cont. for $(x,y) \neq (0,0)$

a direction vctor of $\ y = 3x+4 \qquad x = t, y = 3t+4 \qquad$ is $(1,3)$ so $\mathbf{u}{=}\frac{1}{\sqrt{10}}\,(1,3)$

and by Theorem $D_{\mathbf{u}}f($