

The University of Calgary
Department of Mathematics and Statistics
MATH 349 Handout # 5

1. For $f(x, y) = \frac{xy}{\sqrt{1+x^2}}$ and $\mathbf{g} : \mathbb{R}^2 \rightarrow \mathbb{R}^2, \mathbf{g}(s, t) = (\cos(\pi st), \sin \frac{\pi s}{t})$
 - (a) find ∇f ;
 - (b) using Chain Rule find $\nabla h(0, -1)$ where $h = f \circ \mathbf{g}$
(or $h(s, t) = f(x, y)$ where $x = \cos(\pi st)$ and $y = \sin \frac{\pi s}{t}$)

2. Find an equation of the tangent plane to $z = f(x, y) = \ln(x + y^2)$ at the point $x_0 = 0, y_0 = -1$.

3. For the function $f(x, y) = e^{\sqrt{\frac{y}{x}}}$ find the domain and f_{xx} and f_{xy} .

4. For $f(x, y, z) = \sqrt{2} \sin(\pi xy + x \ln z)$ and $\mathbf{g} : \mathbb{R} \rightarrow \mathbb{R}^3, \mathbf{g}(t) = (\frac{1}{t}, -\frac{1}{t}, \frac{t}{2})$
 - (a) find ∇f ;
 - (b) find $D\mathbf{g}$ or \mathbf{g}'
 - (c) using Chain Rule find $h'(2)$ where $h = f \circ \mathbf{g}$
(or $h(t) = f(x, y, z)$ where $x = \frac{1}{t}, y = -\frac{1}{t}$ and $z = \frac{t}{2}$)

5. In what directions at the point $P(2, 1)$ does the function $f(x, y) = \ln\left(\frac{x}{y} + \frac{y}{x}\right)$ have the rate of change equal to $\frac{3}{10}$? What is the maximum rate of change at that point?

6. Find the rate and the direction of the most rapid decrease of $f(x, y, z) = x^2 z e^y + x z^2$ at the point $P(1, \ln 2, 2)$.

7. Given $f(x, y) = \begin{cases} \frac{xy}{2x^2 + y^2} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{at } (0, 0) \end{cases}$
 - (a) Is f continuous at $(0, 0)$?
 - (b) Find ∇f at $(0, 0)$, if it exists.
 - (c) Find the directional derivative at $(0, 0)$ in the direction of $y = x$ if it exists.
 - (d) Find the directional derivative at $(-1, -1)$ where $\mathbf{u} = \frac{1}{\sqrt{2}}(1, 1)$.