

**FINAL HANDOUT**  
**MATH 349 .**

1. Is the sequence  $a_n = \frac{n^3}{3^n}$  convergent, ult.monotonic,bounded? Find upper and lower bounds.
2. Find the interval of convergence for  $\sum_{n=1}^{\infty} \frac{(3x-1)^n}{3^{3n}\sqrt{n}}$ .
3. Find the Taylor series of  $f(x) = xe^x$  around the centre  $c = -1$ .  
For which  $x$  is the representation valid?
4. For the curve  $c$  given as the intersection of two surfaces  
 $c = \{z = x^2 + y^2\} \cap \{6x - 2y - z = 1\}$ 
  - (a) find a parametrization;
  - (b) find an equation of the tangent line at  $P(0, -1, 1)$ ;
  - (c) set up the integral for evaluating arclength between the points  $P$  and  $R(3, 2, 13)$ . Do not evaluate.
5. Find an equation of the tangent plane to the surface  
 $z = xyx^2$  at the point  $x = 1, y = -1$ .
6. For the function  $f$  defined as follows  
 $f(x, y) = \frac{x^2}{x^2 + 3y^2}$  for  $(x, y) \neq (0, 0)$  and  $f(0, 0) = 0$ 
  - (a) find the gradient  $\nabla f$  at the origin;
  - (b) is  $f$  continuous at the origin? Explain.
7. Find the domain of definition, range and level curves ( $c = 0, 1, -1$ )  
for  $z = \ln \frac{x}{y}$ .
8. Find the directional derivative of  $f(x, y, z) = e^{-y}(x^2 + \cos z)$   
at the point  $A(2, 0, \pi)$  in the direction towards the point  $B(-1, -1, 0)$ .

9. Given  $f(2, -1) = 11, f_x(2, -1) = -7, f_y(2, -1) = 5, f(2, -2) = 9,$   
 $f_x(2, -2) = 3, f_y(2, -2) = -2,$  where all partials are continuous, and  
 $\mathbf{g}(u, v) = (uv^2, \frac{u}{v})$   
 find  $\frac{\partial F}{\partial u}$  at the point  $(2, -1)$  if  $F = f \circ \mathbf{g}$  i.e.  $F(u, v) = f(x, y)$  where  
 $x = uv^2, y = \frac{u}{v}.$

10. The temperature  $T$  at points  $(x, y)$  of the  $xy$ - plane is given by  $T(x, y) = y^2 - 3x^2$
- (a) Draw (find) the isotherms (level curves)  $T = 0, 3, -6.$
  - (b) In what direction should an ant sitting at  $(-1, 2)$  move to cool off as quickly as possible?
  - (c) In what direction(s) is No change in the temperature?

11. Show that the systems of equations

$$xy^2 - z + u^2 = 3$$

$$x^3z + 2y - u = 0$$

$$xu + y - xyz = 3$$

can be solved for  $x, y, z$  as functions of  $u$  around the point  
 $(x, y, z, u) = (1, 1, -1, 1).$

12. Find the Taylor polynomial  $T_2(x, y)$  for the function

$$f(x, y) = \ln(y \cos x)$$
 around the center  $(0, 1).$

13. Show that the equation  $\arcsin(zy) + z^3x + x^2y + 8 = 0$

can be solved for  $z$  as a function of  $x, y$  around the point  $P(-1, 0, 2)$

then find  $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$  at that point..