

Notice to all students:

First read instructions **carefully before doing the assignment**

MAPLE ASSIGNMENT OFFICIAL COVER SHEET

Due Date : NOVEMBER: 25 OR 27 / 2008

(AT END OF YOUR TUTORIAL)

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DO NOT WRITE YOUR NAME ON COVER SHEET
TYPE YOUR NAME ON THE TOP RIGHT HAND CORNER
OF FIRST PAGE OF YOUR ASSIGNMENT

WARNING

COPYING ASSIGNMENT FROM ANOTHER INDIVIDUAL OR
SHARING SOME ONE'S ACCOUNT IS STRICTLY PROHIBITED.

CHEATING / PLAGIARISM IS AN EXTREMELY
SERIOUS ACADEMIC OFFENCE.

ALL CHEATING CASES **WILL BE REPORTED** To
THE DEAN OF YOUR FACULTY

I agree that this paper may be placed at the front of the classroom for
pick-up. Please Circle : Yes or No.

DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 349 MAPLE COMPUTER ASSIGNMENT
Due Date : November: 25 or 27 / 2008
(At End Of Your Tutorial)

1. Use the command `limit` to calculate the limit of the sequence :

$$\left\{ \sqrt[3]{n^3 - 37n^2 - 2n + 17} - \sqrt[3]{n^3 - 10n^2 + 15n - 25} \right\}$$

2. Use the command `sum` to find the sum of the series $\sum_{n=1}^{\infty} a_n$ where

$$a_n = \frac{3n^2 + 5n + 1}{n^4 + 14n^3 + 59n^2 + 70n + 18}$$

3. Use the command `taylor` to find The first five terms of Maclaurin series of $f(x) = \cos(\ln(x + 1))$.
4. Use the commands `taylor` and `convert/polynom` to find Taylor polynomial of degree 4 for the function $g(x) = \arcsin\left(\frac{x}{2}\right)$ about $c = 1$.

5. Find the arc length of the space curve :

$$\vec{r}(t) = 2e^t \vec{i} + e^{2t} \cos(t) \vec{j} + e^{2t} \sin(t) \vec{k} \quad , \quad 0 \leq t \leq \ln(3)$$

6. The position of a particle in space at any time $t \geq 0$ (in seconds) is given by $x(t) = t \frac{3}{2}$, $y(t) = 2t \frac{5}{2}$, $z(t) = t^2$ (in meters). Use the commands `diff` and `fsolve` to find the time t when the particle first reach the speed of 300 m/s.

7. Let $W = \frac{1}{x^2 + y^2 - z^2}$, where $x = e^{-2t}$, $y = e^{-2t} \sin(6s)$, $z = e^{-2t} \cos(6s)$.

(a) Use the commands `diff` and `simplify` to find a simplified expression for $\frac{\partial^2 W}{\partial t \partial s}$.

(b) Use the command `eval` to evaluate $\frac{\partial^2 W}{\partial t \partial s}$ at $t = -\ln(2)$ and $s = \frac{\pi}{24}$.

8. Find the directional derivative of $F(x, y, z) = \ln(4x^2 + 2y^2 + 3z^2)$ at the point $P(-1, 2, 4)$ in the direction of the vector $\vec{a} = (3, -6, -2)$.
9. Use the command `plot3d` to plot the surface $V = \sin(x^5 + y^5)$, $-1 \leq x \leq 1$, $-1 \leq y \leq 1$.

Note : You must display your graph with the following characteristics :

Style : Surface with wireframe or Patch , Axis : None. Do Not Rotate Surface!

NOTES ON MAPLE COMPUTER ASSIGNMENT

(A) WHERE CAN I DO MY ASSIGNMENT ?

You can do your Maple Assignment in Math. Department on the 5th. floor of Mathematical Sciences Building , Rooms MS 515 , MS 521 on a drop in basis After 5:00 PM. , or Room MS 571 on drop basis almost most of the time!

You must check time table posted outside these rooms for availability.

You may print your solutions by directing your file to printing services located at the Microcomputer labs , Elbow Room, ST 142.

However you must have an IT Computer Account. If you don't have an IT account you may fill out an application requesting an account through the campus IT Web site:

<http://www.ucalgary.ca/it>

or simply go to 7th. floor of Math Sciences Building and Apply!

Warning : Using your own PC Is Not Permitted.

You must Save An Electronic Copy On U Of C Computer Just In Case We Need it for Verification or Cheating Investigation.

(B) IMPORTANT INSTRUCTIONS :

Your Assignment must be done on letter size papers. It must also be Stapled and you must have :

The Official Cover Sheet Clearly Displaying :

I. Your U Of C ID Number.

II. Your Tutorial Number (Not Lecture Number!)

III You Must **Print** Your Name On The First
Page Of Your Assignment (Not On Cover Sheet!!)

Hand Written Names Or Solutions Will Not Be Accepted.

The Entire Assignment Will Be Returned Unmarked.

Make sure that you type the problem number say 3 ,7 (b), ... etc.

All problems must be done in Order.

(C) Completed Maple Assignments Must Be Submitted To Your
Tutorial Instructor. Any Assignments Brought to Lectures Or Offices
Will Not Be Marked.

Note :Your Assignment **Will not be Accepted** If Instructions Of
Part (B) **Were Not Strictly Followed**. You Will Receive " F " Grade.

(D) DUE DATE : At the end of your Tutorial :

On Tuesday November 25 / 2008 : For T 01 , T 03 , & T 04.

On Thursday November 27 / 2008 : For T 02 , T05 , & T 06

There are No Extensions !!

(E) NEED HELP FOR A MAPLE COMMAND?

If you need help say for the Maple Command *plot3d* just type the
following: *?plot3d* then press Enter!

Good luck To All.

Maple Help

1.General :

(a) To type a text such as Your Name , A comment , or an Explanation , click on the " T " on the menu bar on top. to resume typing Mathematics click on the " > " on the menu bar on top.

(b) Each mathematical command must starts with a cursor , namely the symbol > and must ends in a semi colon namely the symbol ;

(c) Make sure that each pair of Parentheses : (,) is opened and closed properly.

(d) When typing mathematics , No Spaces Necessary.

(e) At any time : To clear Maple's internal memory use the command " restart ":

Simply type **restart** after the **cursor** > followed by the semi colon ; then press enter.

2.The Basic Operations :

Addition (+) , Subtraction (-) , Multiplication (*) , Division (/) , and Powers (^) .

Now the function $f = \frac{x^3 - 6x - 4}{9 + 5x^{\frac{3}{4}}}$ must be typed in Maple as follows:

> f := (x^3 - 6 * x - 4) / (9 + 5 * x^(3/4)); then press enter.

If your typing is correct you must get : $f := \frac{x^3 - 6x - 4}{9 + 5x^{\frac{3}{4}}}$.

3.Special Notations:

(a) The Square Root Function:

In Maple you may type a square root of an expression in two ways.

For example $\sqrt{2 - 5 \sin(x)}$ may be typed as :

Either $(2 - 5 * \sin(x))^{(1/2)}$, Or $\text{sqrt}(2 - 5 * \sin(x))$

(b) Exponential Functions:

In Maple an exponential function such as e^{1-3x} must be typed as

$$\text{exp}(1 - 3 * x)$$

(c) Logarithmic Functions :

In Maple a logarithmic function such as $\ln(x)$, $\text{Log}_5(x)$ must be typed respectively as :

$$\ln(x) \quad , \quad \log [5](x)$$

Now if you press enter you get :

$$\ln(x) \quad , \quad \frac{\ln(x)}{\ln(5)} \quad (\text{the equivalent of } \log_5(x)).$$

(d) Inverse Trigonometric Functions:

An inverse trigonometric function such as say $\tan^{-1}(x)$ must be typed as $\arctan(x)$.

(e) The Number π must be typed in Maple as Pi (with Capital P)!!

4.The Basic Maple Commands:

(a) The evalf command :

This command is used to evaluate functions :

An Example Find $\ln(4)$,correct to 5 decimal places

`evalf(ln(4),6);` or `evalf[6](ln(4));`

Now press enter you get :

1.38629

(b) The diff command:

This command is used to find the derivative of any order of a function with respect to a specified variable.

Here is an example : Find the derivative of $g(x) = \sin^{-1}(3 - 4x^2) - e^{-\sqrt{2-5x}}$.

`> g(x) := arcsin(3 - 4 * x^2) - exp(-sqrt(2 - 5 * x));`

Now press enter you get :

$$g(x) := -\arcsin(-3 + 4x^2) - e^{(-\sqrt{2-5x})}$$

Let us name the derivative say $h(x)$. To find $h(x)$ we proceed as follows:

`> h(x) := diff(g(x),x);`

Now if you press enter you get:

$$h(x) := -\frac{4x}{\sqrt{-2 + 6x^2 - 4x^4}} - \frac{5}{2} \frac{e^{(-\sqrt{2-5x})}}{\sqrt{2-5x}}.$$

Another example : Find $\frac{d^5y}{dx^5}$ if $y = \ln(2x + 3)$.

We proceed as follows :

`> diff(ln(2 * x + 3),x,x,x,x,x) : OR`

`> diff(ln(2 * x + 3),x$5);`

Now press Enter to get :

$$\frac{768}{(2x + 3)^5}$$

(c) The int command :

The command is used to compute indefinite and definite integrals.

Here is an example : Find (1) $\int x^5 e^{x^2} dx$ (2) $\int_{-3}^1 \frac{1}{\sqrt{55 - 6x - x^2}} dx$

For #1 proceed as follows :

`> y := x^5 * exp(x^2); int(y,x);`

Press enter you get:

$$\frac{1}{2}x^4 e^{(x^2)} - x^2 e^{(x^2)} + e^{(x^2)}$$

No need to name integrand as y but it is recommended for beginners. Example # 2 will be done faster!!

For # 2 proceed as follows :

> int(1 / sqrt(55 - 6 * x - x^2), x = -3..1);

Now press enter you get :

$$\frac{\pi}{6}$$

(d) The fsolve command:

This command is used to solve equations in a specified variable with or without restrictions.

Here are a couple of examples:

(a) Find all critical points of the function $y = x^4 - 20x^3 - 74x^2 - 8x + 10$.

(b) Find all critical points of the function y on the interval $[-1, \infty)$.

For part (a) no restrictions given. Of course we need to solve the equation $y' = 0$.

Proceed as follows:

> y := x^4 - 20 * x^3 - 74 * x^2 - 8 * x + 10;

> z := diff(y,x);

> fsolve(z = 0, x);

Now press enter to get :

$$-2.107337569, -0.05529831490, 17.16263588$$

For part (b) replace the last statement by:

> fsolve(z = 0, x = -1..infinity);

Now press enter to get :

$$-0.05529831490, 17.16263588$$

(e) The solve command:

The solve command solves one or more equations or inequalities for their unknowns.

Here is an example : Find open interval where the function $y = \frac{4x^2 - 8x + 5}{1 - 3x}$ is increasing or decreasing.

We proceed as follows :

> y := (4x^2 - 8 * x + 5)/(1 - 3 * x);

> z := diff(y,x);

Indeed , y is increasing if $z > 0$, and is decreasing if $z < 0$. So we need to solve inequalities.

> solve(z > 0);

Now press Enter to get :

$$y := \frac{4x^2 - 8x + 5}{1 - 3x}$$

$$z := \frac{8x - 8}{1 - 3x} + \frac{3(4x^2 - 8x + 5)}{(1 - 3x)^2}$$

$$\text{Real Range} \left(\text{Open} \left(-\frac{1}{2} \right), \text{Open} \left(\frac{1}{3} \right) \right), \text{Real Range} \left(\text{Open} \left(\frac{1}{3} \right), \text{Open} \left(\frac{7}{6} \right) \right)$$

Next ,

> solve(z < 0);

Now press Enter to get :

$$\text{Real Range} \left(-\infty, \text{Open} \left(-\frac{1}{2} \right) \right), \text{Real Range} \left(\text{Open} \left(\frac{7}{6} \right), \infty \right)$$

(f) The factor command:

This command is used to compute the factorization of a polynomial function with integer , rational , irrational or complex coefficients.

Here is an example : Factor each of the following functions with integer coefficients.

$$(a) f(x) = x^3 + x^2 - 5x + 3 \qquad (b) g(x) = x^4 - 5x^2 - 10x - 6$$

$$(c) h(x) = \frac{3x^3 - 13x^2 + 18x - 8}{x^5 - x^4 - 3x^3 + 3x^2 - 4x + 4}$$

We proceed as follows :

For part (a) :

> factor(x^3 + x^2 - 5 * x + 3);

Now press Enter to get :

$$(x + 3)(x - 1)^2$$

For part (b) :

> factor(x^4 - 5 * x^2 - 10 * x - 6);

Now press Enter to get :

$$(x - 3)(x + 1)(x^2 + 2x + 2)$$

For part (c) :

$$h := (3 * x^3 - 13 * x^2 + 18 * x - 8)/(x^5 - x^4 - 3 * x^3 + 3 * x^2 - 4 * x + 4) ;$$

> factor(h);

Now press Enter to get :

$$\frac{3x - 4}{(x + 2)(x^2 + 1)}$$

(g) The simplify command:

This command is used to simplify your answers.

Here is an example:

Find the derivative of $y = \frac{x^2}{2} \sin^{-1}(x) + \frac{1}{4} \cos^{-1}(x) + \frac{x\sqrt{1-x^2}}{4} - 27$. Simplify your answer.

Proceed as follows:

> y := (x^2/2) * arcsin(x) + arccos(x)/4 + x * sqrt(1 - x^2)/4 ;

z := diff(y,x); simplify(z); OR simplify(%);

Now press enter to get :

$\text{xarc}(\sin(x))$

(h) The limit command :

This command is used to compute limits.

Here is an example :

Determine $\lim_{x \rightarrow 0} \frac{\sin(x) - x}{x^3}$.

Proceed as follows :

> limit((sin(x) - x)/x^3, x = 0);

Now press enter to get

$$-\frac{1}{6}$$

Another Example: Determine $\lim_{x \rightarrow \infty} x^2(1 - \cos(\frac{3}{x}))$.

Proceed as follows :

> limit(x^2 * (1 - cos(3/x)), x = infinity);

Now if you press enter you get :

$$\frac{9}{2}$$

(i) The implicitdiff command.

This command is used to find the derivative $\frac{dy}{dx}$ of a function $y = f(x)$ defined implicitly by a relation in x and y .

Example1 : Assume that the relation $\sin(x + y) = y^2 \cos(x)$ defines y implicitly

as a function of x . Find $\frac{dy}{dx}$.

Proceed as follows :

> g := sin(x + y) = y^2 * cos(x) ; Der := implicitdiff(g, y, x);

Now press enter to get :

$$Der := \frac{\cos(x + y) + y^2 \sin(x)}{-\cos(x + y) + 2y \cos(x)}$$

Example 2 : Find the slope of the tangent line to the curve

$$\sqrt[3]{xy} = 14x + y - 56 \text{ at the point } (2, 32).$$

Proceed as follows :

> u := (x * y)^(1/3) = 14 * x + y - 56 ; v := implicitdiff(u, y, x); eval(v, {x = 2, y = 32}); simplify(%);

Now press enter to get :

$$\frac{-320}{23}$$

(j) The (two dimensional) plot command.

This command is used to plot the graph of a function of a single variable.

Example (a) sketch graph of $y = 2x^3 - 7x + 23$, $x \in [-4, 4]$.

Proceed as follows:

```
> y := 2 * x^3 - 7 * x + 23;
```

```
> plot (y, x = -4..4);
```

Now press enter, your graph is displayed in the specified domain.

There are other interesting features that you may want to explore your self.

Just type `?plot` then press enter you get a page of help!

Note: for graphing a piecewise function a certain procedure must be followed (depending on Maple version you have!!).

To sketch a piecewise function a "Procedure" may be used as illustrated in the example below:

Sketch the graph of $y = \begin{cases} x^2 & \text{if } -3 < x < 2 \\ x + 2 & \text{if } 2 \leq x \leq 3 \\ 5 \sin\left[\frac{\pi(x-2)}{2}\right] & \text{if } 3 < x \leq 4 \end{cases}$

```
> y := proc(x) if x < 2 then x^2 elif x >= 2 and x <= 3 then x + 2 else
```

```
5*sin(Pi * (x - 2)/2) end if end proc ;
```

```
> plot (y, -3..4);
```

Now press enter, your graph is displayed!!

(k) The (3 dimensional) plot3d command.

This command is used to plot the graph of a function of a two independent variables.

Example (a) sketch the graph of the surface $z = x^2 + 5y^2$, $x \in [-3, 3]$, $y \in [-6, 6]$.

Proceed as follows :

```
> z := x^2 + 5 * y^2 ; plot3d(z, x = -3..3, y = -6..6, style = patch) ;
```

Now press Enter, your graph is displayed in the specified domain.

There are other interesting features that you may want to explore your self.

(l) The diff command: Revisited !!

This command is used to find the partial derivative (of any order) of a function of several variables.

as well as derivative of a vector valued function .Here is an example :

(a) Find $\frac{\partial^5 z}{\partial x^3 \partial y^2}$ if $z = y^2 \sin(2x + y^3)$.

(b) Find the velocity $\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$ if $\vec{r}(t) = (t^2, 4 \cos(3t), e^{7t})$, $t \in \mathbb{R}$

We proceed as follows :

Part (a) :

> `diff(y^2 * sin(2 * x + y^3), y, y, x, x, x);` **OR**

> `diff(y^2 * sin(2 * x + y^3), y$2, x$3);`

Now press Enter to get :

$$-16 \cos(2x + y^3) + 144y^3 \sin(2x + y^3) + 72y^6 \cos(2x + y^3)$$

Part (b)

> `with(VectorCalculus);` $\vec{v} := \text{diff}(\langle t^2, 4 \cos(3t), e^{7t} \rangle, t);$

Now press Enter to get

$$\vec{v} = \langle 2t, -12 \sin(3t), 7 e^{7t} \rangle$$

Note : To find the *speed* $s = \|\vec{v}\|$, you may use the *Norm* Command :

`with(LinearAlgebra);` $s := \text{Norm}(\vec{v}, 2, \text{conjugate} = \text{false});$

Now press Enter to get

$$s = \sqrt{4t^2 + 144 \sin^2(3t) + 49 e^{14t}} .$$

(m) The (Double Integral) Doubleint command.

The command is used to compute definite (or indefinite) double integrals.

Here is an example : Evaluate $\int_0^1 \int_0^{y^2} \sin(\pi y^3) dx dy$.

We proceed as follows:

> `with(student);` $F := \text{Doubleint}(\sin(\text{Pi} * y^3), x = 0..y^2, y = 0..1);$ `value(F);`

Now press Enter to get :

$$\frac{2}{3\pi}$$

(n) The (Triple Integral) Tripleint command

The command is used to compute definite (or indefinite) triple integrals.

Here is an example : Evaluate $\int_0^3 \int_0^{2(3-y)/3} \int_0^{6-2y-3z} x^2 dx dz dy$.

We proceed as follows:

> `with(student);` $L := \text{Tripleint}(x^2, x = 0..6 - 2 * y - 3 * z, z = 0..2 * (3 - y)/3, y = 0..3);$ `value(L);`

Now press Enter to get :

$$\frac{108}{5}$$

(o) The sum command :

The *sum* command can be used to find the exact sum of an infinite series. If there is no exact expression for the sum , then the *sum* command just return the series!

However the " *evalf* " command may be able to approximate the sum of the series.

Here is an example : Find the sum of

$$(a) \sum_{n=0}^{\infty} \frac{(\ln(3))^{2n}}{(2n)!}$$

$$(b) \sum_{n=3}^{\infty} \frac{n^2}{2^n + 1}$$

We proceed as follows :

For part (a) :

> sum ((ln(3))^(2 * n)/(2 * n)!, n = 0..infinity);

Now press Enter to get :

$$\frac{5}{3}$$

For part (b) :

> S := sum (n^2/(2^n + 1), n = 3..infinity);

Now press Enter to get

$$\sum_{n=3}^{\infty} \frac{n^2}{2^n + 1}$$

We just got the series back! Hence *sum* command can not find exact sum.

Let us instead find an approximate value of the sum *S*.

We proceed as follows :

> evalf(S);

Now press Enter to get :

$$4.279463462$$

(P) The *taylor* command & The *convert / polynom* command

The *taylor* command compute the order *n* *Taylor Series* expansion of a function *f* with about the point $x = c$. Note that the symbol at end, namely $O((x - c)^{n+1})$ is a reminder of the terms not shown, constituting of the rest of the series.

Note that for approximation purposes, we need the *Taylor Polynomial* of degree *n* about $x = c$.

You can not just subtract the last term $O((x - c)^{n+1})!!$, you must use the *convert / polynom* command :

Here is an example :

(a) Find the First 5 terms of the *Taylor series* of $f(x) = \sqrt{2x + 1}$ about $c = 4$.

(b) Find the *Taylor Polynomial* of degree 3 of $g(x) = \arctan(x)$ about $c = 1$.

(c) Find the first three terms of *Maclurin series* of $h(x) = \sec(x) + \frac{1}{1+x}$

We proceed as follows :

For part (a) :

> taylor (sqrt(2 * x + 1), x = 0, 5);

Now press Enter to get :

$$3 + \frac{1}{3}(x - 4) - \frac{1}{54}(x - 4)^2 + \frac{1}{486}(x - 4)^3 - \frac{5}{17496}(x - 4)^4 + O((x - 4)^5)$$

For part (b) : Note : To get Polynomial of degree 3, you need to first find *Taylor series* of order $3 + 1 = 4$

> $S := \text{taylor}(\arctan(x), x = 1, 4); P := \text{convert}(S, \text{polynom});$

Now press Enter to get :

$$\frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{1}{12}(x-1)^3 + O((x-1)^4)$$

$$\frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{1}{12}(x-1)^3$$

For part (c) : Note : For Maclaurin Series $c = 0$.

> $\text{taylor}(\sec(x) + 1/(1+x), x = 0, 3);$

Now press Enter to get :

$$2 - x + \frac{3}{2}x^2 + O(x^3)$$

(q) The Directional Derivative command :

The *Directional Derivative* command computes the directional derivative of a function f evaluated at a point P in the given direction \vec{v} .

Here is an example :

Find the directional derivative of $T(x, y, z) = x^2 + xyz + 3$ at the point $P(2, -1, 3)$

in the direction of $\vec{v} = (1, 4, -7)$.

We proceed as follows :

> $\text{with}(\text{Student}[\text{MultivariateCalculus}]); \text{DirectionalDerivative}(x^2 + x * y * z, [x, y, z] = [2, 1, -3], [-1, 2, -2]);$

Now press Enter to get :

$$-\frac{17}{3}$$

Refer to Maple Help Work Sheet For All Examples Presented In This Help Sheet. Good Luck To All.

MAPLE WORKSHEET

YOUR NAME :Yousry Elsabrouty

#1

> $f := (x^3 - 6x - 4) / (9 + 5x^{(3/4)});$

$$f := \frac{x^3 - 6x - 4}{9 + 5x^{(3/4)}} \quad (1)$$

#2

> $(2 - 5 \sin(x))^{(1/2)};$

$$\sqrt{2 - 5 \sin(x)} \quad (2)$$

> $\text{sqrt}(2 - 5 \sin(x));$

$$\sqrt{2 - 5 \sin(x)} \quad (3)$$

#3

> $\text{exp}(1 - 3x);$

$$e^{(1 - 3x)} \quad (4)$$

> $\ln(x);$

$$\ln(x) \quad (5)$$

> $\log[5](x);$

$$\frac{\ln(x)}{\ln(5)} \quad (6)$$

#4

> $\text{arctan}(x);$

$$\text{arctan}(x) \quad (7)$$

#5

> $\text{evalf}(\ln(4), 6);$

$$1.38629 \quad (8)$$

> $\text{evalf}[6](\ln(4));$

$$1.38629 \quad (9)$$

#6

> $g(x) := \arcsin(3 - 4x^2) - \exp(-\text{sqrt}(2 - 5x));$

$$g(x) := -\arcsin(-3 + 4x^2) - e^{(-\sqrt{2 - 5x})} \quad (10)$$

> $h(x) := \text{diff}(g(x), x);$

$$h(x) := -\frac{4x}{\sqrt{-2 + 6x^2 - 4x^4}} - \frac{5}{2} \frac{e^{(-\sqrt{2 - 5x})}}{\sqrt{2 - 5x}} \quad (11)$$

#7

> diff(ln(2 x + 3), x, x, x, x, x);

$$\frac{768}{(2x + 3)^5} \quad (12)$$

OR,

> diff(ln(2 x + 3), x\$5);

$$\frac{768}{(2x + 3)^5} \quad (13)$$

#8

> y:=x^5*exp(x^2);

$$y := x^5 e^{(x^2)} \quad (14)$$

> int(y, x);

$$\frac{1}{2} x^4 e^{(x^2)} - x^2 e^{(x^2)} + e^{(x^2)} \quad (15)$$

#9

> int(1/sqrt(55-6*x-x^2), x=-3..1);

$$\frac{\pi}{6} \quad (16)$$

#10

> y:=x^4-20*x^3-74*x^2-8*x+10;

$$y := x^4 - 20x^3 - 74x^2 - 8x + 10 \quad (17)$$

> z:=diff(y, x);

$$z := 4x^3 - 60x^2 - 148x - 8 \quad (18)$$

> fsolve(z=0, x);

$$-2.107337569, -0.05529831490, 17.16263588 \quad (19)$$

with the restriction given we get:

> fsolve(z=0, x=-1..infinity);

$$-0.05529831490, 17.16263588 \quad (20)$$

#11

> y := (4x^2 - 8x + 5) / (1 - 3x); z := diff(y, x);

$$y := \frac{4x^2 - 8x + 5}{1 - 3x}$$

$$z := \frac{8x - 8}{1 - 3x} + \frac{3(4x^2 - 8x + 5)}{(1 - 3x)^2} \quad (21)$$

> solve(z > 0);

$$\text{RealRange}\left(\text{Open}\left(-\frac{1}{2}\right), \text{Open}\left(\frac{1}{3}\right)\right), \text{RealRange}\left(\text{Open}\left(\frac{1}{3}\right), \text{Open}\left(\frac{7}{6}\right)\right) \quad (22)$$

> solve(z < 0);

$$\text{RealRange}\left(-\infty, \text{Open}\left(-\frac{1}{2}\right)\right), \text{RealRange}\left(\text{Open}\left(\frac{7}{6}\right), \infty\right) \quad (23)$$

#12

> factor(x³ + x² - 5x + 3);

$$(x + 3)(x - 1)^2 \quad (24)$$

#13

> factor(x⁴ - 5x² - 10x - 6);

$$(x - 3)(x + 1)(x^2 + 2x + 2) \quad (25)$$

#14

> h := $\frac{(3x^3 - 13x^2 + 18x - 8)}{(x^5 - x^4 - 3x^3 + 3x^2 - 4x + 4)}$;

$$h := \frac{3x^3 - 13x^2 + 18x - 8}{x^5 - x^4 - 3x^3 + 3x^2 - 4x + 4} \quad (26)$$

> factor(h);

$$\frac{3x - 4}{(x + 2)(x^2 + 1)} \quad (27)$$

#15

> y := (x²/2)*arcsin(x) + arccos(x)/4 + x*sqrt(1-x²)/4;

$$y := \frac{1}{2}x^2 \arcsin(x) + \frac{1}{4} \arccos(x) + \frac{x\sqrt{1-x^2}}{4} \quad (28)$$

>

> z := diff(y, x);

$$z := x \arcsin(x) + \frac{x^2}{4\sqrt{1-x^2}} - \frac{1}{4\sqrt{1-x^2}} + \frac{\sqrt{1-x^2}}{4} \quad (29)$$

> simplify(z);

$$x \arcsin(x) \quad (30)$$

#16

> limit((sin(x)-x)/x³, x=0);

$$\frac{-1}{6} \quad (31)$$

```
> limit(x^2*(1-cos(3/x)),x=infinity);
```

$$\frac{9}{2}$$

(32)

```
#17
```

```
> g:=sin(x+y)=y^2*cos(x); Der:=implicitdiff(g,y,x);
```

$$g := \sin(x + y) = y^2 \cos(x)$$

$$Der := \frac{\cos(x + y) + y^2 \sin(x)}{-\cos(x + y) + 2 y \cos(x)}$$

(33)

```
> u:=(x*y)^(1/3)=14*x+y-56; v:=implicitdiff(u,y,x); eval(v,{x=2,y=32}); simplify(%);
```

$$u := (x y)^{(1/3)} = 14 x + y - 56$$

$$v := -\frac{y - 42 (x y)^{(2/3)}}{x - 3 (x y)^{(2/3)}}$$

$$-\frac{32 - 42 \cdot 64^{(2/3)}}{2 - 3 \cdot 64^{(2/3)}}$$

$$\frac{-320}{23}$$

(34)

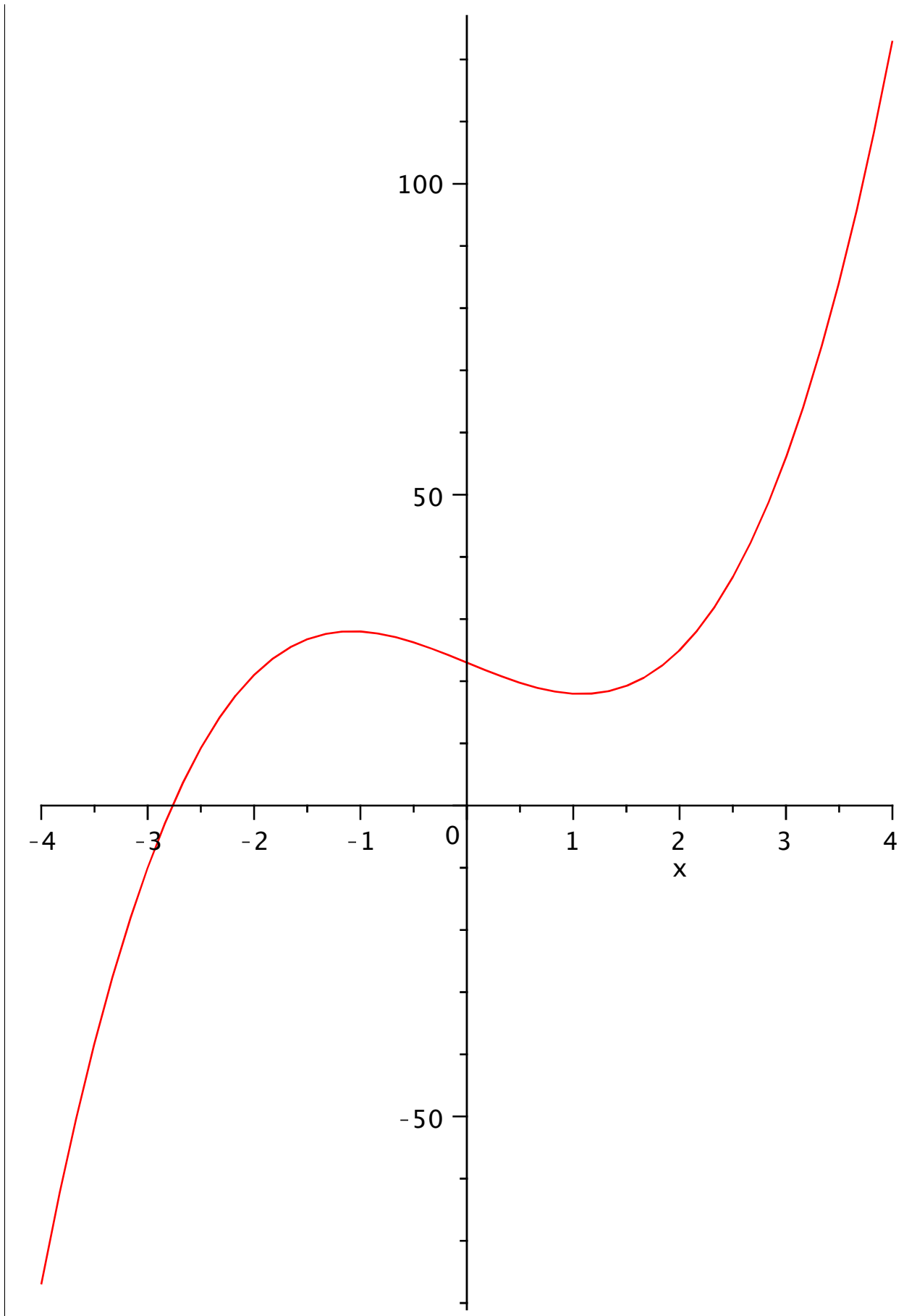
```
#18
```

```
> y:=2*x^3-7*x+23;
```

$$y := 2 x^3 - 7 x + 23$$

(35)

```
> plot(y,x=-4..4);
```



#19

```
> y:=proc(x)if x<2 then x^2 elif x>=2 and x<=3 then x+2 else 5*sin  
  (Pi*(x-2)/2)end if end proc;
```

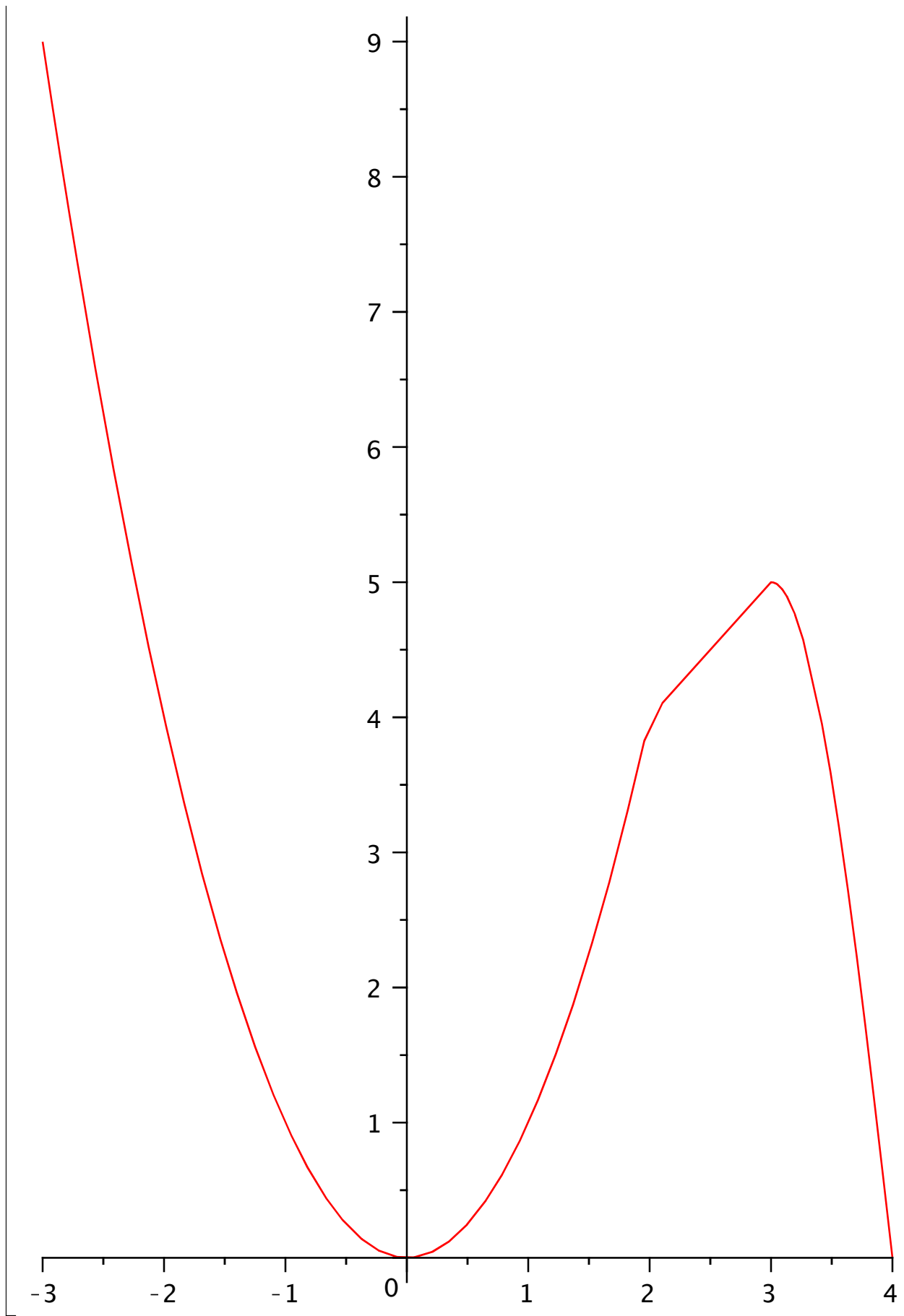
```
y:=proc(x)
```

(36)

```
  if x < 2 then x^2 elif 2 <= x and x <= 3 then x + 2 else 5 * sin(1/2 * pi * (x - 2)) end if
```

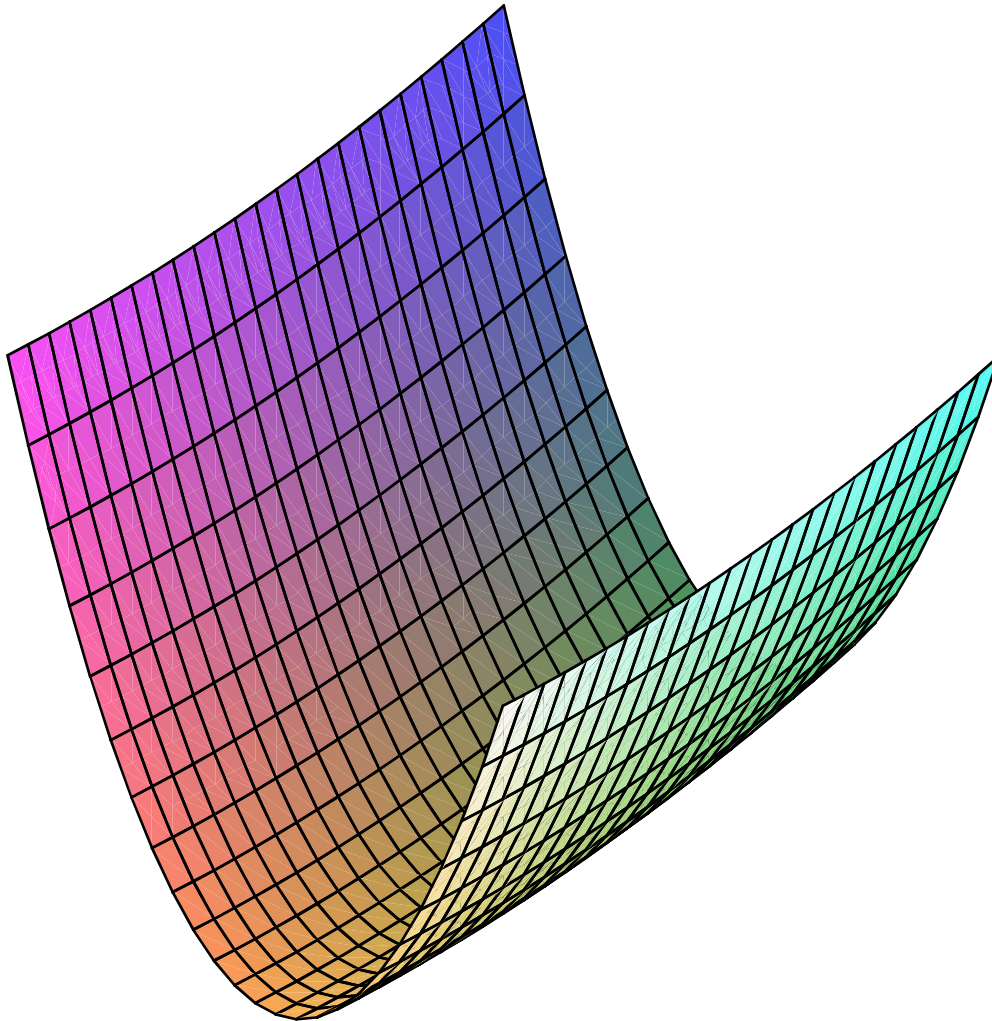
```
end proc
```

```
> plot(y, -3..4);
```



#20

```
> z:=x^2+5*y^2;plot3d(z,x=-3..3,y=-6..6,style=patch);  
z:=x^2+5*y^2
```



```
>
```

```
>
```

#21

```
> diff(y^2 sin(2x+y^3), y$2, x$3);  
-16 cos(2x+y^3) + 144 y^3 sin(2x+y^3) + 72 y^6 cos(2x+y^3)
```

(37)

```
>
```

#22

```
> with(VectorCalculus); v:=diff(<t^2, 4 cos(3t), e^7t>, t);  
Warning, these names have been rebound: `&x`, CrossProduct,  
DotProduct, Norm, Normalize  
[&x, `*`, `+`, `-', `:`, <, >, <|>, AddCoordinates, ArcLength, BasisFormat, Binormal,  
CrossProd, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProd,
```

DotProduct, Flux, GetCoordinateParameters, GetCoordinates, Gradient, Hessian, Jacobian, Laplacian, LineInt, MapToBasis, Nabla, Norm, Normalize, PathInt, PrincipalNormal, RadiusOfCurvature, ScalarPotential, SetCoordinateParameters, SetCoordinates, SurfaceInt, TNBFrame, Tangent, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, Wronskian, diff, evalVF, int, limit, series]

$$v := 2te_x - 12\sin(3t)e_y + 7e^{(7t)}\ln(e)e_z \quad (38)$$

Which is the same as

$$(2t, -12\sin(3t), 7(e^{7t}).)$$

>

#23

```
> with(LinearAlgebra); s := Norm(v, 2, conjugate = false);
Warning, these names have been rebound: `&x`, CrossProduct,
DotProduct, Norm, Normalize
[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm,
BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, Column,
ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix,
ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation,
CrossProduct, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix,
Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors,
Equal, ForwardSubstitute, FrobeniusForm, GaussianElimination, GenerateEquations,
GenerateMatrix, GetResultDataType, GetResultShape, GivensRotationMatrix,
GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm,
HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite,
IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, LA_Main,
LUDecomposition, LeastSquares, LinearSolve, Map, Map2, MatrixAdd, MatrixExponential,
MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower,
MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular,
Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent,
Pivot, PopovForm, QRDecomposition, RandomMatrix, RandomVector, Rank,
RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation,
RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues,
SmithForm, SubMatrix, SubVector, SumBasis, SylvesterMatrix, ToeplitzMatrix, Trace,
Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle,
VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]
```

(39)

$$s := \sqrt{4t^2 + 144 \sin(3t)^2 + 49(e^{7t})^2 \ln(e)^2} \quad (39)$$

>

#24

> `with(student); F := Doubleint(sin(π y3), x=0..y2, y=0..1); value(F);`

Warning, the previous binding of the name D has been removed and it now has an assigned value

[D, Diff, Doubleint, Int, Limit, Lineint, Product, Sum, Tripleint, changevar, completesquare, distance, equate, integrand, intercept, intparts, leftbox, leftsum, makeproc, middlebox, middlesum, midpoint, powsubs, rightbox, rightsum, showtangent, simpson, slope, summand, trapezoid]

$$F := \int_0^1 \int_0^{y^2} \sin(\pi y^3) \, dx \, dy = \frac{2}{3} \frac{1}{\pi} \quad (40)$$

>

#25

> `restart; with(student); L := Tripleint(x2, x=0..6-2*y-3*z, z=0..2*(3-y)/3, y=0..3); value(L);`

[D, Diff, Doubleint, Int, Limit, Lineint, Product, Sum, Tripleint, changevar, completesquare, distance, equate, integrand, intercept, intparts, leftbox, leftsum, makeproc, middlebox, middlesum, midpoint, powsubs, rightbox, rightsum, showtangent, simpson, slope, summand, trapezoid]

$$L := \int_0^3 \int_0^{2-\frac{2}{3}y} \int_0^{6-2y-3z} x^2 \, dx \, dz \, dy = \frac{108}{5} \quad (41)$$

>

26

>

> `sum((ln(3))^(2*n)/(2*n!), n=0..infinity);`

$$\frac{5}{3} \quad (42)$$

27


```
> S := sum(n^2 / (2^n + 1), n = 3 .. ∞);
```

$$S := \sum_{n=3}^{\infty} \frac{n^2}{2^n + 1} \quad (43)$$

```
> evalf(S);
```

$$4.279463462 \quad (44)$$

```
# 28
```

```
> taylor(sqrt(2*x + 1), x = 4, 5);
```

$$3 + \frac{1}{3} (x-4) - \frac{1}{54} (x-4)^2 + \frac{1}{486} (x-4)^3 - \frac{5}{17496} (x-4)^4 + O((x-4)^5) \quad (45)$$

```
>
```

```
# 29
```

```
> P := taylor(arctan(x), x = 1, 4); P := convert(P, polynom);
```

$$P := \frac{1}{4} \pi + \frac{1}{2} (x-1) - \frac{1}{4} (x-1)^2 + \frac{1}{12} (x-1)^3 + O((x-1)^4)$$

$$P := \frac{1}{4} \pi + \frac{1}{2} x - \frac{1}{2} - \frac{1}{4} (x-1)^2 + \frac{1}{12} (x-1)^3 \quad (46)$$

```
>
```

```
# 30
```

```
> taylor(sec(x) + 1/(x+1), x = 0, 3);
```

$$2 - x + \frac{3}{2} x^2 + O(x^3) \quad (47)$$

```
>
```

```
# 31
```

```
> with(Student[MultivariateCalculus]); DirectionalDerivative(x^2 + x*y*z, [x, y, z] = [2, 1, -3], [-1, 2, -2]);
```

```
[ApproximateInt, ApproximateIntTutor, CenterOfMass, ChangeOfVariables, CrossSection, CrossSectionTutor, Del, DirectionalDerivative, DirectionalDerivativeTutor, FunctionAverage, Gradient, GradientTutor, Jacobian, LagrangeMultipliers, MultiInt, Nabla, Revert, SecondDerivativeTest, SurfaceArea, TaylorApproximation, TaylorApproximationTutor]
```

$$-\frac{17}{3} \quad (48)$$

```
>
```