The University of Calgary
Department of Mathematics and Statistics
MATH 349 Lecture 01/ 02
Quiz # 1R

Fall 2008

Name:______________________________**I.D.#:**____________________

1. Find the limit of the sequence $a_n = n - 5^n$. [3]

2. Is the sequence$\{a_n\}$ ultimately monotonic ,bounded,alternating, convergent? Explain; find the limit, an upper and a lower bounds

   (a) $a_n = \dfrac{(-1)^n n}{\ln(n^2+1)}$;

   (b) $a_n = \dfrac{n^2}{1+n^3}$ [7]

SOLUTION

For 1)

$\lim\limits_{n\to\infty}(n-5^n) = \text{"}\infty - \infty\text{"} = \lim\limits_{n\to\infty} 5^n\left(\dfrac{n}{5^n} - 1\right) = \infty\,(-1) = -\infty$ since $\lim\limits_{x\to\infty}\dfrac{x}{5^x} = (L'H.R.) = \lim\limits_{x\to\infty}\dfrac{1}{5^x(\ln 5)} = 0$

For2a)

the sequence $a_n = \dfrac{(-1)^n n}{\ln(n^2+1)}$ is **alternating thus NOT monotonic**

for n even: $\lim\limits_{n\to\infty}\dfrac{n}{\ln(n^2+1)} = \text{"}\dfrac{\infty}{\infty}\text{"}(l.H.R) = \lim\limits_{x\to\infty}\dfrac{1}{\dfrac{2x}{x^2+1}} = \lim\limits_{x\to\infty}\dfrac{x^2+1}{2x} =$ (again)$= \lim\limits_{x\to\infty}\dfrac{2x}{2} = +\infty$

for n odd: $\lim\limits_{n\to\infty}\dfrac{-n}{\ln(n^2+1)} = \text{"}\dfrac{\infty}{\infty}\text{"}(l.H.R) = \lim\limits_{x\to\infty}\dfrac{-1}{\dfrac{2x}{x^2+1}} = \lim\limits_{x\to\infty} -\dfrac{x^2+1}{2x} =$ (again)$= \lim\limits_{x\to\infty} -\dfrac{2x}{2} = -\infty$

yhus the sequence is**divergent neither bounded above nor below**

For 2b)

$a_n = \dfrac{n^2}{1+n^3} > 0 \qquad \lim\limits_{n\to\infty}\dfrac{n^2}{1+n^3} = \lim\limits_{n\to\infty}\dfrac{\dfrac{1}{n}}{\dfrac{1}{n^3}+1} = 0$ **convergent** for monotonicity : $f'(x) = \left(\frac{x^2}{1+x^3}\right)' = \frac{2x(1+x^3) - x^2 \cdot 3x^2}{(1+x^3)^2} = \frac{2x - x^4}{(1+x^3)^2} = \frac{x(2-x^3)}{(1+x^3)^2} < 0$

for $n \geq 2$ so the sequence is **ult. decreasing**

the best **upper bound** is $a_1 = \frac{1}{2} > a_2 = \frac{4}{9}$

the best **lower bound** is 0.