The University of Calgary
Department of Mathematics and Statistics
MATH 349-01/02
Quiz # 4R

Fall,2008

I.D.#:__________________ **Name:**

1. For $f(x,y) = \ln\sqrt{\dfrac{x}{y}}$
   (a) sketch the domain of f;find the range;
   (b) sketch the level curves of f for any c.

[3]

2. Find $\displaystyle\lim_{(x,y)\to(-1,0)} \frac{\sin(xy+y)}{2(x+1)^2+3y^2}$, if it exists. [3]

3. Find both partial derivatives of $f(x,y) = \sin^2(xy^2)$ at the point $\left(\dfrac{\pi}{4}, -1\right)$. [4]

Solution For 1)
domain :$x/y > 0$,both positive or both negative
first and third quadrants without axes
$c = \ln\sqrt{\dfrac{x}{y}} = \dfrac{1}{2}\ln\dfrac{x}{y} \qquad e^{2c} = \dfrac{x}{y} \to y = e^{-2c}x$
LINES THROUGH THE ORIGIN WITHOUT IT
slope is $m = e^{-2c}$
the range is $(-\infty, +\infty)$

For 2)
define $f(x,y) = \dfrac{\sin(xy+y)}{2(x+1)^2+3y^2}$around the point $(-1,0)$
then for $y \neq 0, x = -1 \qquad f(-1,y) = \dfrac{0}{3y^2} = 0$
for $x \neq -1, y = 0 \qquad f(x,0) = \dfrac{0}{2(x+1)^2} = 0$
let's try a line through the point $(-1,0) \qquad y = m(x+1) \qquad$ for $x \neq -1$ and $m \neq 0$
$f(x,m(x+1)) = \dfrac{\sin(m(x+1)^2)}{2(x+1)^2+3m^2(x+1)^2} = \dfrac{\sin(m(x+1)^2)}{m(x+1)^2}\cdot\dfrac{m}{(2+3m^2)}$
as $x \to -1 \qquad f(x,m(x+1)) \to \dfrac{m}{(2+3m^2)} \neq 0$ (e.g.for $m = 1$)
since $\lim_{h\to 0}\frac{\sin\ h}{h} = 1 \qquad$ thus **the limit DNE-does not exist.**
ALSO
define $X = x+1$ and change the limit to $\displaystyle\lim_{(X,y)\to(0,0)}\frac{\sin(Xy)}{X^2+3y^2}$
with lines $y = mX$

For 3)

partials

$$f_x(x,y) = \left[(\sin(xy^2)^2\right]' = 2\sin(x^2y)\cdot\frac{\partial}{\partial x}\Big(\sin(xy^2)\Big) = 2\sin(x^2y)\cdot\cos(xy^2)\cdot\frac{\partial}{\partial x}(xy^2) =$$

$$= 2\sin(x^2y)\cdot\cos(xy^2)\cdot y^2 = y^2\sin(2x^2y)$$

at $x = \frac{\pi}{4}$ $\quad y = -1$ $\quad$ using $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\quad f_x = 1$

similarily

$$f_y(x,y) = 2\sin(x^2y)\cdot\cos(xy^2)\cdot\frac{\partial}{\partial y}(xy^2) = 2\sin(x^2y)\cdot\cos(xy^2)\cdot(2xy) = 2xy\sin(2xy^2)$$

at $x = \frac{\pi}{4}$ $\quad y = -1$ $\quad$ using $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\quad f_y = -\frac{\pi}{2}.$