The University of Calgary
Department of Mathematics and Statistics
MATH 349-01/02
Quiz # 4T

Fall 2008

Name:_____I.D.#:_____

1. For $f(x,y)=\sqrt{x^2+2x+y^2}$
   (a) sketch the domain of f;
   (b) find the range
   (c) sketch the level curves of f for $c=0,1,-1,...$; [3]
2. Find the partials for $f(x,y)=\ln(x^2+y\sqrt{x})$ in the domain. [3]
3. Find $\lim\limits_{(x,y)\to(0,0)}\dfrac{x^3}{x^2+y^2}$ if it exists. [4]

Solution.

For 1)

for the domain $x^2+2x+y^2\geq 0 \qquad x^2+2x+1+y^2\geq 1$

$D=\{(x,y):(x+1)^2+y^2\geq 1\}$ the region outside the circle
with the centre at $(-1,0)$ and radius $R=1$
level curves: $c=\sqrt{x^2+2x+y^2}$ only for $c\geq 0$
we get circles $(x+1)^2+y^2=1+c^2$
with the centre at $(-1,0)$ and radius $R=\sqrt{1+c^2}$
thus the range is $[0,+\infty)$.

For 2)

$\dfrac{\partial f}{\partial x}=\dfrac{2x+\dfrac{y}{2\sqrt{x}}}{x^2+y\sqrt{x}} \qquad \dfrac{\partial f}{\partial y}=\dfrac{\sqrt{x}}{x^2+y\sqrt{x}}$ for $x>0$ and $y>-x^{\frac{3}{2}}$.

For3)

define $g(x,y)=\dfrac{x^3}{x^2+y^2} \qquad g(0,y)=\dfrac{0}{y^2}=0$ for $y\neq 0$,

for $x\neq 0 \qquad g(x,0)=\dfrac{x^3}{x^2}=x\to 0$ as $x\to 0$

so the limit could be 0 ;to prove it estimate

for $x\neq 0$ since $x^2+y^2\geq x^2 \qquad \left|\dfrac{x^3}{x^2+y^2}-0\right|=\dfrac{|x|^3}{x^2+y^2}\leq\dfrac{|x|^3}{x^2}=|x|\to 0$

by Squ.Th. the limit is 0.