The University of Calgary
Department of Mathematics and Statistics
MATH 349 Lecture 01/ 02
Quiz # 1T

Fall 2008

Name:______________________________**I.D.#:**__________________

1. Find the limit of the sequence $a_n = (2+n^2)^{\frac{1}{n}}$. [3]

2. Is the sequence$\{a_n\}$ ultimately monotonic ,bounded,alternating, convergent? Explain, find the limit, an upper and a lower bound

(a) $a_n = \dfrac{(-1)^n\, 2^n}{3^n+1}$;

(b) $a_n = \dfrac{n}{1+\sqrt{n}}$ [7]

SOLUTION

For1)

$a_n = (2+n^2)^{\frac{1}{n}} = e^{\frac{1}{n}\ln(2+n^2)}$

$$\lim_{x\to\infty}\frac{\ln(2+x^2)}{x} = (L'H.R.\text{twice})\lim_{x\to\infty}\frac{\frac{2x}{2+x^2}}{1} = \lim_{x\to\infty}\frac{2}{2x} = 0$$

so $\lim\limits_{n\to\infty} a_n = e^0 = 1$.

For 2a)

it an alternating sequence thus **NOT momotonic** since $a_n = \pm\dfrac{2^n}{3^n+1}$

we can estimate $|a_n| \le \dfrac{2^n}{3^n+1} \le (\dfrac{2}{3})^n \to 0$ since it is a geom sequ, with $r = \dfrac{2}{3} < 1$

therefore by Squ.Th also $\{a_n\}$ is **convergent** to 0

lower bond ..$-1 \le \dfrac{-2^n}{3^n+1} \le a_n \le \dfrac{2^n}{3^n+1} \le 1$.. upper bound

For 2b)

$a_n = \dfrac{n}{1+\sqrt{n}} \ge 0$..lower bound $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty}\dfrac{n}{1+\sqrt{n}} = \lim\limits_{n\to\infty}\dfrac{\sqrt{n}}{\frac{1}{\sqrt{n}}+1} = +\infty$ or L'H.R.

so NO upper bound and the sequ. is **divergent to** ∞

for monotonicity $f'(x) = \left(\dfrac{x}{1+\sqrt{x}}\right)' = \dfrac{1+\sqrt{x} - x\cdot\frac{1}{2\sqrt{x}}}{(1+\sqrt{x})^2} = \dfrac{1+\frac{1}{2}\sqrt{x}}{(1+\sqrt{x})^2} > 0$

so the sequ. is **increasing**