The University of Calgary
Department of Mathematics and Statistics
MATH 349-01/02 Quiz # 3R

Fall 2008

Name:______________________________**I.D.#:**____________________

1. Is the series $\sum_{n=1}^{\infty} (-1)^n \sin \frac{1}{n}$ absolutely or conditionally convergent or divergent?

   Explain. [3]

2. Find the centre,radius and interval of convergence of power series

   $\sum_{n=1}^{\infty} \frac{(3x+1)^n}{\sqrt{n}}$ [3]

3. Express $f(x) = \frac{1}{(2x-1)^2}$ in powers of $(x+1)$.

   On what interval is the representation valid? [4]

   Solutions

For1)

First ,let's try abs.convergence $\sum_{n=2}^{\infty} \left|(-1)^n \sin \frac{1}{n}\right| = \sum_{n=2}^{\infty} \sin \frac{1}{n}$

$a_n = \sin \frac{1}{n}$ is equivalent to $b_n = \frac{1}{n}$ since $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin \frac{1}{n}}{\frac{1}{n}} = \lim_{h\to 0} \frac{\sin h}{h} = 1$

and $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$ so the series is divergent by Limit Comp.Test.

For conditionally convergence let's investigate the sequence $\{a_n\}$:

limit is 0 and a_n is decreasing since $\left(\sin \frac{1}{x}\right)' = \cos \frac{1}{x} \cdot \left(\frac{-1}{x^2}\right) < 0$ for $x \geq 1$

thus by Alt.Test the original series is **cond.convergent**

For 2)

$$\sum_{n=1}^{\infty} \frac{(3x+1)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{3^n(x+\frac{1}{3})^n}{\sqrt{n}} \qquad c = -\frac{1}{3} \qquad \text{and} \qquad a_n = \frac{3^n}{\sqrt{n}}$$

for the radius

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{3^{n+1}}{\left(\sqrt{n+1}\right)} \cdot \frac{\sqrt{n}}{3^n} = \left(\sqrt{\frac{n}{n+1}}\right) 3 \to 3 \text{ as } n \to \infty, \qquad \text{so } R = \frac{1}{3}$$

and the series is **absolutely convergent f**or $x \in (c-R, c+R) = \left(-\frac{2}{3}, 0\right)$

now for $x = 0$

we get the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \infty \qquad (p = \frac{1}{2} < 1)$

for $x=-\dfrac{2}{3}$ we get

$\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ and it is **conditionally convergent** by Alt.Test,since $\dfrac{1}{\sqrt{n}}\searrow 0$

and together the series is **convergent** on $\left[-\dfrac{2}{3},1\right)$

For 3)

set $x+1=t$ $\quad x=t-1$ $\quad$ for $x\neq\dfrac{1}{2}$

$$f\left(x\right)=\frac{1}{\left(2x-1\right)^2}=\frac{1}{\left(2t-3\right)^2}=\frac{1}{3^2}\cdot\frac{1}{\left(1-\frac{2}{3}t\right)^2}=$$

$$=\frac{1}{3^2}\sum_{n=1}^{\infty}n\ \frac{2^{n-1}}{3^{n-1}}\ t^{n-1}=\sum_{n=1}^{\infty}\frac{2^{n-1}}{3^{n+1}}\ n\ \left(x+1\right)^{n-1}$$

for $-1<\frac{2(x+1)}{3}<1,$ so $-3<2x+2<3$, and finally $-\frac{5}{2}<x<\frac{1}{2}$.

(using $\displaystyle\sum_{n=1}^{\infty}n\ r^{n-1}=\frac{1}{\left(1-r\right)^2}$ if $-1<r<1$ for $r=\dfrac{2}{3}t$)