The University of Calgary
Department of Mathematics and Statistics
MATH 349-01/02 Quiz # 3T

Fall 2008

Name:______________________________**I.D.#:**__________________

1. Is the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ absolutely or conditionally convergent or divergent?

   Explain. [3]

2. Find the centre,radius and interval of convergence of power series

   $\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n3^n}$ [3]

3. Express $f(x) = \frac{1}{(3x+1)^2}$ in powers of $(x-1)$.

   On what interval is the representation valid? [4]

   Solutions

For1)

First ,let's try abs.convergence $\sum_{n=2}^{\infty} \left| \frac{(-1)^n}{\ln n} \right| = \sum_{n=2}^{\infty} \frac{1}{\ln n}$

for $n \geq 2$ the coef.are positive and since $\ln n < n$

$a_n = \frac{1}{\ln n} > b_n = \frac{1}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$ so the series is **divergent** by Comp.Test.

For conditionally convergence let's investigate the sequence $\{a_n\}$:

limit is 0 and $\{a_n\}$ is decreasing $= \frac{1}{incr.pos.}$

thus by Alt.Test the original series is **cond.convergent**

For 2)

$\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n3^n} = \sum_{n=1}^{\infty} \frac{2^n (x+\frac{1}{2})^n}{n3^n}$

the centre is $c = -\frac{1}{2}$ and $a_n = \frac{2^n}{n3^n}$

for the radius

$\left| \frac{a_{n+1}}{a_n} \right| = \frac{2^{n+1}}{(n+1)\,3^{n+1}} \cdot \frac{n3^n}{2^n} = \left(\frac{n}{n+1}\right) \frac{2}{3} \to \frac{2}{3}$ as $n \to \infty$, so $R = \frac{3}{2}$

and the series is **absolutely convergent f**or $x \in (c-R, c+R) = (-2, 1)$

now for $x = 1$

we get the series $\sum_{n=1}^{\infty} \frac{3^n}{n} \frac{1}{3^n} = \sum_{n=1}^{\infty} \frac{1}{n} = \infty-$harmonic series

for $x = -2$ we get

$\sum_{n=1}^{\infty} \frac{(-3)^n}{n3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

and it is alt.harmonic series **conditionally convergent** by Alt.Test,
therefore the original series is cond.covergent at $x = -2$
and together the series is **convergent** on $[-2, 1)$

For 3)

set $x - 1 = t \qquad x = 1 + t \qquad$ for $x \neq -\dfrac{1}{3}$

$$\frac{1}{(3x+1)^2} = \frac{1}{(4+3t)^2} = \frac{1}{4^2} \cdot \frac{1}{\left(1 + \frac{3}{4}t\right)^2} =$$

$$= \frac{1}{4^2} \sum_{n=1}^{\infty} (-1)^{n-1}\, n\, \frac{3^{n-1}}{4^{n-1}} t^{n-1} = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{3^{n-1}}{4^{n+1}}\, n\, (x-1)^{n-1}$$

for $-1 < \frac{3(x-1)}{4} < 1$, so $-4 < 3x - 3 < 4$, and finally $-\frac{1}{3} < x < \frac{7}{3}$.

(using $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\, n\, r^{n-1} = \frac{1}{(1+r)^2}$ if $-1 < r < 1$, for $r = \dfrac{3}{4}t$)