MATH349 – Assignment 1

Due: Wednesday May 25

1. Write the first five terms of each of the following sequences:
   (a) $\left\{ \dfrac{3+(-1)^n \sin(\pi n/2)}{n!} \right\}$.
   (b) $\{a_n\}$ where $a_n = \dfrac{1}{2 \cdot 4 \cdot 6 \cdots (2n)}$.
   (c) $\{x_n\}$ where $x_n = \dfrac{1}{(2n) \cdot x_{n-1}}$ and $x_1 = 2$.

2.(a) Give an example of a sequence which is bounded but is not convergent.
(b) Give an example of a sequence which is increasing and converges to 0.

3. Find sequences $\{x_n\}$ and $\{y_n\}$ such that $x_n - y_n$ converges to 0, $\sum x_n$ diverges and $\sum y_n$ converges.

4. Show that the sequence $\{u_n\}$ where $u_n = \dfrac{\sqrt{n}}{n+1}$ is
   (a) monotonically decreasing
   (b) bounded above
   (c) bounded below
   (d) has a limit

5. Consider the series $\displaystyle\sum_{i=2}^{\infty} \left(\frac{2}{3}\right)^i$.
   (a) Write the first four partial sums of this series.
   (b) Series of this type were given a name in class. What is it?
   (c) Show that this series is convergent.
   (d) Find the value of the series.

(Warning: Note the series starts at $i = 2$.)

6. Show that $\displaystyle\sum_{n=1}^{\infty} \frac{4}{(4n-1)(4n+3)} = \frac{1}{3}$.

7.(a) State the Ratio Test.
(b) Use the Ratio Test to determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^3}$ is convergent.

8.(a) State the Integral Test.
(b) Use the Integral Test to determine whether $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$ is convergent.

9. Explain why the following argument is false.

> If $S = 1-1+1-1+1-1+\cdots$, then $S = 1-(1-1)-(1-1)-(1-1)-\cdots = 1$.
> Also, $S = (1-1)+(1-1)+(1-1)+\cdots = 0$. Hence $1 = 0$.

Bonus question: Show that the sequence $\{x_n\}$ defined by

$$x_{n+1} = \frac{4x_n + 2}{x_n + 3} \quad \text{and} \quad x_1 = 3$$

converges to 2.