MATH349 – Assignment 2

Due: Wednesday June 1

1. Determine the centre, radius of convergence and interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{n(x-1)^n}{2^n(3n-1)}$$

2. Consider the power series $\displaystyle\sum_{n=2}^{\infty} \frac{(x+2)^n}{\log n}$.
   (a) Write down the centre and the first five coefficients.
   (b) Determine the radius of convergence.
   (c) Examine the two endpoints of the interval of convergence.

3.(a) Give the Taylor series for $\sin x$ and $\cos x$ about the point 0.
   (b) Find the power series expansion for $\sin x \cos x$ about the point 0. (Use (a).)
   (c) Find the Taylor series expansion for $\sin 2x$ about the point 0.
   (d) Confirm that $2 \sin x \cos x = \sin 2x$.

4.(a) Use the binomial theorem to show that

$$\frac{1}{\sqrt{1-t^2}} = 1 + \frac{1}{2}t^2 + \frac{1 \cdot 3}{2 \cdot 4}t^4 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}t^6 + \dots$$

   (b) Use (a) and $\sin^{-1} x = \displaystyle\int_0^x \frac{dt}{\sqrt{1-t^2}}$ to show

$$\sin^{-1} x = x + \frac{1}{2}\,\frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4}\,\frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\,\frac{x^7}{7} + \dots$$

   (c) In what region is this expansion valid?
   (d) Conclude that

$$\frac{\pi}{2} = 1 + \frac{1}{2 \cdot 3} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7} + \dots$$

5. Find $\displaystyle\int_0^1 \frac{1 - \cos x}{x}\, dx$ correct to 3 decimal places.

6.(a) Give the power series expansion for $\log(1+x)$ about the point 0.
   (b) Determine the power series expansions about 0 for $x - \log(1+x)$ and $x \log(1+x)$.
   (c) Use (b) to show

$$\lim_{x \to 0} \left(\frac{1}{\log(1+x)} - \frac{1}{x} \right) = \frac{1}{2}$$

7. Determine the coefficient of x^4 in the power series expansion about 0 of $\dfrac{1}{(1+x+x^2)^4}$.

8. Use the binomial theorem to show the following

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n-1} + \binom{n}{n}$$

Bonus question: Show that

$$1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \frac{1}{11} - \frac{1}{6} + \dots = \frac{3 \log 2}{2}$$