MATH349 – Assignment 3

Due: Wednesday June 8

1. For the points:
$$P = (1, 2, -3)\,, \quad Q = (2, -3, 4)\,, \quad R = (-3, 1, 2)\,.$$
   (a) Find the line passing through P and Q.
   (b) Give the equation of the plane passing through all three points P, Q, R.

2. Find all vectors $\mathbf{v}$ in $\mathbb{R}^3$ which satisfy the equation
$$(-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \times \mathbf{v} = \mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$$

3. Given the three vectors
$$\mathbf{u} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}\,, \quad \mathbf{v} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}\,, \quad \mathbf{w} = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}\,.$$
   (a) Determine $\mathbf{u} + \mathbf{w}$.
   (b) Determine $|\mathbf{v}|$.
   (c) Find the projection of $\mathbf{u} + \mathbf{w}$ in the direction of $\mathbf{v}$.

4. If $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$, is it true that $\mathbf{v} = \mathbf{w}$?

5. For the vectors $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ determine
$$\big|(\mathbf{a} + \mathbf{b}) \bullet (\mathbf{a} - \mathbf{b})\big|$$

6. Find a vector parallel to the line of intersection of the two planes given by
$$2x - y + z = 1 \quad \text{and} \quad 3x + y + z = 2$$

7. Consider the point $P = (3, -1, 2)$.
   (a) Determine the distance between P and $(1, 4, -1)$.
   (b) Determine the normal vector to the plane $x - 4y + 2z = 1$.
   (c) Determine the minimal distance between P and the plane $x - 4y + 2z = 1$.

8. Consider the function $f\colon \mathbb{R} \to \mathbb{R}^2$ given by $f(t) = (2t, e^t)$.
   (a) Determine $f(0)$, $f(1)$ and $f(-1)$.
   (b) Is f linear?

9. Form all possible composite functions from the four functions below,
$$e\colon \mathbb{R} \to \mathbb{R}^2\,, \quad f\colon \mathbb{R}^3 \to \mathbb{R}\,, \quad g\colon \mathbb{R}^2 \to \mathbb{R}^5\,, \quad h\colon \mathbb{R}^4 \to \mathbb{R}^3\,.$$
   Also give the domain and codomain for each composite function.

Bonus question: If A is an $n \times n$ matrix such that $A^3 = 0$, show that $I - A$ is invertible.