MATH349 – Assignment 5

Due: Wednesday June 22

1. Determine the Frenet frames (i.e. $\hat{\mathbf{T}}$, $\hat{\mathbf{N}}$, $\hat{\mathbf{B}}$) for the curve

$$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$$

at the point $(1, 1, 1)$.

2. Determine the tangent plane, normal vector and normal line for the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = e^{2xy} + 3x^3y^2$$

at the point $(1, 2)$.

3. Draw the graphs of the following functions

(a) $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by $(x, y) \mapsto 4 - x^2$.
(b) $g\colon \mathbb{R}^2 \to \mathbb{R}$ given by $(x, y) \mapsto x^2 + y^2 - 2y$.

4. Draw the level curves of the following functions

(a) $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by $(x, y) \mapsto (x + 1)(y + 3)$.
(b) $g\colon \mathbb{R}^2 \to \mathbb{R}$ given by $(x, y) \mapsto x^2/y$.

5. Find all first partial derivatives of $h\colon \mathbb{R}^3 \to \mathbb{R}$ where $h(x, y, z) = x^2y^4z^7$. Evaluate each of them at the point $(1, 0, -1)$.

6. Compute the matrix of first partial derivatives for the function

$$\begin{aligned} &f\colon \mathbb{R}^2 \to \mathbb{R}^4 \\ &(x, y) \mapsto (x + y, x - y, 2x + y^2, y) \end{aligned}$$

7. For the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

determine $f_x(0, 0)$, $f_y(0, 0)$, $f_{xx}(0, 0)$, $f_{xy}(0, 0)$, $f_{yx}(0, 0)$ and $f_{yy}(0, 0)$.

8. Consider the function $F\colon \mathbb{R}^3 \to \mathbb{R}$ given by $F(x, y, z) = z\sin(y/x)$ and the transformation

$$x = 3r^2 + 2s\ , \quad y = 4r - 2s^3\ , \quad z = 2r^2 - 3s^2\ .$$

Determine $\dfrac{\partial F}{\partial r}$ and $\dfrac{\partial F}{\partial s}$.

9. Find the derivative matrix and the Jacobian of the transformation $S\colon \mathbb{R}^3 \to \mathbb{R}^3$ where

$$(r, \theta, \phi) \mapsto (r\sin\theta\sin\phi, r\cos\theta\sin\phi, r\cos\phi)$$

The co-ordinates (r, θ, ϕ) are known as the spherical co-ordinates for $\mathbb{R}^3$.

10. Show that the function

$$\begin{aligned} &g\colon \mathbb{R}^3 \to \mathbb{R} \\ &(x, y, z) \mapsto \frac{1}{\sqrt{x^2 + y^2 + z^2}} \end{aligned}$$

satisfies Laplace's partial differential equation, namely

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

Bonus question: Use the ε–δ definition of a limit to show that

$$\lim_{(x,y)\to(2,1)} (3x + y^2) = 7$$