University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Math 349 Spring 2006

Worksheet 1 (Sequences)

1. Write the first five terms of the sequence given:

a. $\left\{\frac{n}{2n+1}\right\}_{n=1}^{\infty}$

b. $\{a_n\}_{n=1}^{\infty}$, $a_n = \frac{1\cdot3\cdot5\cdots(2n+1)}{n!}$

c. $\left\{\sin\frac{n\pi}{2}\right\}_{n=1}^{\infty}$

d. $a_1 = 1$, $a_{n+1} = \frac{1}{1+a_n}$

2. Find a formula for the general term, a_n, of the given sequence, in each case, assuming that the pattern of the first few terms continues:

a. $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots\right\}$

b. $\{4, 1, 4, 1, \ldots\}$

c. $\{1, 4, 7, 10, \ldots\}$

d. $\left\{\frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \frac{6}{49}, \ldots\right\}$

e. $\{1, 2, 4, 7, 11, \ldots\}$

f. $\left\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\right\}$

g. $\{1, 3, 8, 16, 27, 41, \ldots\}$

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3. Determine whether the sequence given converges or diverges. If it converges, find the limit.

a. $\left\{\frac{n+1}{3n+1}\right\}_{n=1}^{\infty}$

b. $\left\{\frac{n}{1+\sqrt{n}}\right\}_{n=1}^{\infty}$

c. $\left\{\frac{n^2-1}{n^2+1}\right\}_{n=1}^{\infty}$

d. $\left\{(-1)^n \frac{n^2}{n^3+1}\right\}_{n=1}^{\infty}$

e. $\left\{\frac{1}{5^n}\right\}_{n=1}^{\infty}$

f. $\left\{\frac{\cos^2 n}{2^n}\right\}_{n=1}^{\infty}$

g. $\left\{n^{-1/n}\right\}_{n=1}^{\infty}$

h. $\left\{\frac{\ln n^2}{n}\right\}_{n=1}^{\infty}$

i. $\left\{\frac{\sin n}{\sqrt{n}}\right\}_{n=1}^{\infty}$

j. $\left\{\tan^{-1}\left(\frac{2n}{2n+1}\right)\right\}_{n=1}^{\infty}$

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k. $\left\{\sin \frac{n\pi}{2}\right\}_{n=1}^{\infty}$

l. $\{\ln(n+1) - \ln n\}_{n=1}^{\infty}$

m. $\{(1+3n)^{1/n}\}_{n=1}^{\infty}$

n. $\left\{\left(\sqrt{n+1} - \sqrt{n}\right)\sqrt{n+\frac{1}{2}}\right\}_{n=1}^{\infty}$

o. $\left\{\frac{1}{n^2} + \frac{2}{n^2} + \ldots + \frac{n}{n^2}\right\}_{n=1}^{\infty}$

p. $\left\{\frac{\pi^n}{3^n}\right\}_{n=1}^{\infty}$

q. $\left\{\frac{n!}{2^n}\right\}_{n=1}^{\infty}$

r. $\left\{\frac{n!}{(n+2)!}\right\}_{n=1}^{\infty}$

4. In each case determine whether the given sequence is increasing, decreasing, or not monotonic:

a $\left\{\frac{1}{3n+5}\right\}$

b $\left\{\cos \frac{n\pi}{2}\right\}$

c $\left\{\frac{\pi^n}{e^n}\right\}$

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d $\left\{ \sin \frac{1}{n} \right\}$

e $\left\{ \frac{1}{5^n} \right\}$

f $\left\{ \frac{(n!)^2}{(2n)!} \right\}$

5. Show that the sequence $\left\{ \sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots \right\}$ converges to 2.

6. A sequence $\{a_n\}$ is given by $a_1 = \sqrt{2}, \quad a_{n+1} = \sqrt{2 + a_n}, \quad n \geq 1.$

   a. By induction or otherwise, show that $\{a_n\}$ is increasing and bounded above by 3.

   b. State why the sequence converges and determine $\lim_{n \to \infty} a_n$.

7. Repeat question 6 for the sequence $\{a_n\}$ defined by
$a_1 = 1, \quad a_{n+1} = 3 - \frac{1}{a_n}, \quad n \geq 1.$

8. A sequence $\{a_n\}$ is defined by $a_1 = 1, \quad a_{n+1} = \frac{1}{1 + a_n}$ for all $n \geq 1$. Assuming that $\{a_n\}$ is convergent, find its limit.

9. Show that the sequence $\{a_n\}$ defined by $a_n = \sqrt{n^p}, \quad p \in \mathbb{R}$ converges to 1.

10. Prove that if $\lim_{n \to \infty} |a_n| = 0$, then $\lim_{n \to \infty} a_n = 0$. [Hint: Use the Squeeze theorem].