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1. Give examples of divergent sequences $\{a_n\}$, and $\{b_n\}$ so that

i. $\{ a_n + b_n \}$ is divergent.

ii. $\{ a_n - b_n \}$ is divergent.

iii. $\{ a_n b_n \}$ is divergent.

iv. $\left\{ \dfrac{a_n}{b_n} \right\}$ is divergent.

v. $\{ a_n + b_n \}$ is convergent.

vi. $\{ a_n - b_n \}$ is convergent.

vii. $\{ a_n b_n \}$ is convergent.

viii. $\left\{ \dfrac{a_n}{b_n} \right\}$ is convergent.

2. In each case, show that the services converges and find its sum, s:

i $\displaystyle\sum_{n=0}^{\infty} \left(\frac{5^n}{8^n} \right)$

ii $\displaystyle\sum_{n=1}^{\infty} \left[\sin\left(\frac{1}{n}\right) - \sin\left(\frac{1}{n+1}\right) \right]$

iii $\displaystyle\sum_{n=0}^{\infty} x^n, \qquad |x| < 1$

iv $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(n+1)(n+2)}$

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v $\left\{-\frac{81}{100} + \frac{9}{10} - 1 + \frac{10}{9} + \ldots\right\}$

vi $\sum_{k=2}^{\infty} \left(\frac{1}{k^2 - 1} - \frac{7}{10^{k-1}} \right)$

vii $\sum_{j=1}^{\infty} (-1)^{j-1} \frac{3^{2j}}{2^{3j+1}}$

viii $\sum_{k=2}^{\infty} \ln\left(1 - \frac{1}{k^2}\right)$

3. Find values of x for which the given series converges and find the sum of the series for those values of x

i $\sum_{n=0}^{\infty} \left(\frac{x^n}{5^n} \right)$

ii $\sum_{n=1}^{\infty} 2^n \sin^n x$

iii $\sum_{n=0}^{\infty} \left(\frac{1}{x^n} \right)$

iv $\sum_{n=0}^{\infty} (2x + 1)^{2n}$

v $\sum_{n=1}^{\infty} \left(\frac{x^n}{e^{n+1}} \right)$

vi $\sum_{n=0}^{\infty} (2x - 3)^n$

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4. Write the number

a $0.\overline{15} = 0.15151515...$ as a quotient of integers.

b $2.\overline{312} = 2.312312312312...$ as a quotient of integers.

5. Determine whether the p-series given below converges or diverges

i $\sum_{n=1}^{\infty}\left(\frac{2}{n^{1/3}}\right)$

ii $\sum_{n=1}^{\infty}\left(\frac{1}{n^{1.0001}}\right)$

iii $\sum_{n=1}^{\infty}\left(\frac{1}{n^{\pi}}\right)$

iv $\sum_{n=1}^{\infty}\left(n^{-0.99}\right)$

v $\sum_{n=1}^{\infty}\left(n^{(1-\sqrt{2})}\right)$

vi $\sum_{n=1}^{\infty}\left(\frac{1}{n\sqrt{n}}\right)$

vii $\sum_{n=1}^{\infty}\left(\frac{2}{n\sqrt{n}}+\frac{3}{n^{3}}\right)$

viii $\sum_{n=1}^{\infty}\left(\frac{1}{n^{1/x-1}}\right), \quad \frac{1}{2} < x < 1$

ix $\sum_{n=1}^{\infty}\left(\frac{1}{n^{7/5}}\right)$

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x $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^{\sec x}}\right), \quad 0 < x < \frac{\pi}{2}$

6. Apply the divergence test to show that either the series diverges or the test fails (is inconclusive):

a $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+5}\right)$

b $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n}{3n^2+5}\right)$

c $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2}{3(n+1)(n+2)}\right)$

d $\displaystyle\sum_{k=1}^{\infty} \left(1+\frac{1}{k}\right)^k$

e $\displaystyle\sum_{n=1}^{\infty} (1+n)^{1/n}$

f $\displaystyle\sum_{n=1}^{\infty} \tan^{-1} n$

g $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

h $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{\pi}{2n}\right)$

I $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt{n^2+4n+1}-n\right)$

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j $\sum_{k=1}^{\infty} \ln k$

k $\sum_{n=1}^{\infty} \left(\frac{n}{e^{n}}\right)$

l $\sum_{n=1}^{\infty} \left(\frac{e^{n} - e^{-n}}{e^{n} + e^{-n}}\right)$

7. Use the Integral test to determine whether the series given converges or diverges:

a $\sum_{n=1}^{\infty} \left(n\, e^{-n^2}\right)$

b $\sum_{n=2}^{\infty} \left(\frac{1}{n \ln n}\right)$

c $\sum_{n=1}^{\infty} \left(\frac{\sin \frac{1}{n}}{n^2}\right)$

d $\sum_{n=1}^{\infty} \left(sech^2\, n\right)$

e $\sum_{n=1}^{\infty} \left(\frac{1}{n \ln n\, (\ln(\ln n))^p}\right), \quad 0 < p < 1$

f $\sum_{k=2}^{\infty} \left(\frac{1}{k^p \ln k}\right), \quad p > 1$

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8. Use the ratio test to determine whether the given series converges or diverges. If the test fails, say so:

a $\sum_{n=1}^{\infty} \left(\frac{n^3}{3^n} \right)$

b $\sum_{k=1}^{\infty} \left(\frac{1}{k!} \right)$

c $\sum_{k=1}^{\infty} \left(\frac{k}{2^k} \right)$

d $\sum_{n=1}^{\infty} \left(\frac{n^n}{n!} \right)$

e $\sum_{n=1}^{\infty} \left(\frac{|x|^n}{n!} \right), \quad x \in \mathbb{R}$

f $\sum_{n=1}^{\infty} \left(\frac{1 \cdot 3 \cdot 5 \cdots (3n+1)}{3 \cdot 6 \cdot 9 \cdots (3n)} \right)$

g $\sum_{n=1}^{\infty} \left(\frac{2 \cdot 4 \cdot 6 \cdots (2n)}{2 \cdot 5 \cdot 8 \cdots (3n-1)} \right)$

h $\sum_{n=1}^{\infty} \left(\frac{n^3}{n^4+1} \right)$

I $\sum_{n=1}^{\infty} \left(\frac{2^n}{3^n+1} \right)$

j $\sum_{n=1}^{\infty} \left(\frac{(2n)!}{2^n (n!)^2} \right)$

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k $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n!\ 10^n}{3^n}\right)$

9. Use the root test to determine whether the given series converges or diverges. If the test fails, say so:

a $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2n+3}{3n+2}\right)^n$

b $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{2}{n}\right)^{n^2}$

c $\displaystyle\sum_{k=1}^{\infty}\left(\frac{1}{k^k}\right)$

d $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{5^n}\right)$

e $\displaystyle\sum_{k=1}^{\infty}\left(1+e^{-k}\right)^k$

f $\displaystyle\sum_{n=1}^{\infty}\left(\frac{e^n}{(n+1)^n}\right)$

g $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{2n-1}\right)^{2n}$

h $\displaystyle\sum_{n=1}^{\infty}\left(e^n\left(\frac{n}{1+n}\right)^{n^2}\right)$

I $\displaystyle\sum_{n=2}^{\infty}\frac{1}{(\ln n)^n}$

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10. Use the comparison or limit comparison test to determine whether the given series converges or diverges:

a $\displaystyle\sum_{n=3}^{\infty} \frac{\ln n}{n}$

b $\displaystyle\sum_{n=1}^{\infty} \frac{5}{2n^2 + 4n + 3}$

c $\displaystyle\sum_{k=1}^{\infty} \frac{9}{\sqrt{k} + 1}$

d $\displaystyle\sum_{n=1}^{\infty} \frac{3 + \cos n}{3^n}$

e $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{1 + 3^n}$

f $\displaystyle\sum_{n=1}^{\infty} \frac{7}{3^n + 1}$

g $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 - n + 2}{(n^{10} + n^7 + 15)^{1/4}}$

h $\displaystyle\sum_{n=1}^{\infty} \frac{n + 2}{(n + 1)(n + 4)(n + 11)}$

I $\displaystyle\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^5}$

j $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n\sqrt{n}}$

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k $\displaystyle\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^3+4}}$

l $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n(n+1)}$

11. Use the alternating series test to determine whether the given series converges or diverges:

a $\displaystyle\sum_{n=3}^{\infty} (-1)^n \frac{\ln n}{n}$

b $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n-1} n}{\ln n}$

c $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/n}}$

d $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} e^{-n}$

e $\displaystyle\sum_{k=1}^{\infty} (-1)^{k-1} \frac{k+1}{2k+1}$

f $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}$

g $\displaystyle\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{\pi}{n}\right)$

h $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{\pi}{2n}\right)$

I $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{2^n}$

j $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^{3/4}}$ [Hint: Show first that the series is alternating]

k $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{n+1}{n}\right)^n$

l $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}, \quad p > 0$

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12. Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

a $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$

b $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2 + 1}$

c $\sum_{n=1}^{\infty} \frac{5^n (n + 1)}{n \cdot 3^{2n}}$

d $\sum_{n=1}^{\infty} \frac{\sin 2n}{n^2}$

e $\sum_{n=1}^{\infty} \frac{(-n)^n}{5^{2n+3}}$

f $\sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{6}\right)}{n\sqrt{n}}$

g $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n + 2}{3n - 1}\right)^n$

h $\sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{2}\right)$

I $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

j $\sum_{k=1}^{\infty} \frac{k \cos(k\pi)}{k^2 + 1}$

k $\sum_{n=1}^{\infty} \frac{8 - n^3}{n!}$

l $\sum_{n=1}^{\infty} \frac{n^2 \sin(3n)}{(1.1)^n}$

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13. Use any test to determine whether the series given converges or diverges:

a $\sum_{n=1}^{\infty} \left(\frac{\sqrt{n}}{n^2 + 1} \right)$

b $\sum_{n=1}^{\infty} \cos n$

c $\sum_{n=1}^{\infty} \left(n^2 e^{-n^3} \right)$

d $\sum_{n=1}^{\infty} \frac{\sin 2n}{n^2}$

e $\sum_{k=1}^{\infty} k^{-1.7}$

f $\sum_{n=1}^{\infty} (-1)^n \frac{n}{(n + 1)(n + 2)}$

g $\sum_{k=1}^{\infty} \frac{1}{1 + \left(\frac{3}{\pi} \right)^k}$

h $\sum_{n=1}^{\infty} \frac{2^n}{(2n + 1)!}$

I $\sum_{n=1}^{\infty} (-\pi)^n$

j $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$

k $\sum_{n=1}^{\infty} \frac{\tan \left(\frac{1}{n} \right)}{n}$

l $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$

m $\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$