University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Math 349 Spring 2006

Worksheet 9

1. In each case calculate the derivatives indicated:

a $\left(\frac{\partial y}{\partial x}\right)_u$ if $xyuv = 1$, *and* $x + y + u + v = 0$

b $\left(\frac{\partial x}{\partial y}\right)_z$ if $x^2 + y^2 + z^2 + w^2 = 1$, *and* $x + 2y + 3z + 4w = 2$

c $\left(\frac{\partial y}{\partial z}\right)$ if $e^{yz} - x^2 z \ln(y) = \pi$

2. Show that the system of equations:

$$xy^2 + zu + v^2 = 3$$

$$x^3 z + 2y - uv = 2$$

$$xu + yv - xyz = 1$$

can be solved for x, y, z as functions of u and v near to the point P where $(x,y,z,u,v) = (1,1,1,1,1,)$ and find $\left(\frac{\partial y}{\partial u}\right)_v$ at $(u, v)) = (1, 1)$.

3. Show that the equations

$$xe^y + uz - \cos v = 2$$

$$u \cos(y) + x^2 v - yz^2 = 1$$

can be solved for u and v as functions of x, y, z near to the point P where $(x, y, z) = (2, 0, 1)$ *and* $(u, v) = (1, 0)$, and find $\left(\frac{\partial u}{\partial z}\right)_{x,y}$ at $(x, y, z) = (2, 0, 1)$.

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4. Verify that $\dfrac{\partial(u,v)}{\partial(x,y)} = \dfrac{1}{\dfrac{\partial(x,y)}{\partial(u,v)}}$ if $x = u^3 + v^3, \quad y = uv - v^3$.

5. If $F(x, y, z) = 0$ show that $\left(\dfrac{\partial x}{\partial y}\right)_z \left(\dfrac{\partial y}{\partial z}\right)_x \left(\dfrac{\partial z}{\partial x}\right)_y = -1$.

Assume that all derivatives of F involved exist and are continuous on some domain D.