

The University of Calgary
Department of Mathematics and Statistics
MATH 353 FINAL HANDOUT

1. Sketch the region of integration and evaluate

$$\int_{-1}^0 \left(\int_{-1}^{\sqrt[3]{y}} \frac{dx}{x^4 + 1} \right) dy.$$

2. Find all points on the sphere $x^2 + y^2 + z^2 = 36$ closest to $P(1, 2, 2)$.
 3. Express the integral

$$\iint_D \frac{x + y}{x^2 + y^2} dx dy$$

where D is the region above the line $x + y = 2$ and inside the circle $x^2 + y^2 = 4$

- (a) as iterated integrals in cartesian coordinates;
 (b) as iterated integrals in polar coordinates

then evaluate only once.

4. Evaluate $\iiint_B \frac{z dV}{\sqrt{x^2 + y^2}}$ where $B = \{(x, y, z); 1 \leq z \leq 4 \text{ and } z \geq x^2 + y^2\}$.

5. Find the surface area of S

where S is the part of $z = \sqrt{3x^2 + 3y^2}$ below the plane $x + z = 4$.

6. Find $\oint_c \mathbf{F} \cdot d\mathbf{s}$

where $\mathbf{F} = (y^3 x + \cos(x^2), e^{y^2} + \sin(\pi x))$ and c is boundary of the triangle T from $(0, 2)$ to $(2, 2)$ to $(2, 0)$ and back to $(0, 2)$.

7. Show that for any smooth vector field $\mathbf{F}(x, y)$ and any smooth real-valued function $\phi(x, y)$

$$\text{div}(\phi \mathbf{F}) = \text{grad} \phi \bullet \mathbf{F} + \phi \text{div} \mathbf{F} \quad \nabla \cdot (\phi \mathbf{F}) = \nabla \phi \cdot \mathbf{F} + \phi (\nabla \cdot \mathbf{F}).$$

8. Evaluate $\oint_c \mathbf{F} \cdot d\mathbf{s}$ where $\mathbf{F} = (x^2 + y, y^3 - x, z^4)$

and c is given as $\{x^2 + y^2 = 4\} \cap \{2x - 3y + z = 2\}$ oriented positively.

9. Find the flux of $\mathbf{F} = (x^2, y^2, z^2)$ outward

from the closed surface $S = \{x^2 + y^2 + 4(z - 1)^2 = 4\}$.

10. Evaluate $\int_c \mathbf{F} \bullet d\mathbf{s}$ where $\mathbf{F} = (y, x, 2z)$ and
 $c = \{z = 2xy\} \cap \{x^2 + y^2 = 2\}$ from $A(-1, 1, -2)$ to $B(1, 1, 2)$.
11. For the vector field $\mathbf{F}(x, y, z) = (\arctan z(x^2 + y^2), \ln(1 + y^2 + z^2), y e^{xyz})$
find $\text{div } \mathbf{F}$ and $\text{curl } \mathbf{F}$ in the domain.