The University of Calgary
Department of Mathematics and Statistics
MATH 353 FINAL HANDOUT

1. For $f(x,y) = x^2(y+1)^3 + y^2$show that $(0,0)$ is a local minimum,then decide if it is also absolute min.

   Explain.

2. Sketch the region of integration and evaluate
$$\int_{-1}^{0}\left(\int_{-1}^{\sqrt[3]{y}} \frac{dx}{x^4+1}\right)dy.$$

3. Find all points on the sphere $x^2+y^2+z^2=36$ closest to $P(1,2,2)$.

4. Express the integral
$$\iint_D \frac{x+y}{x^2+y^2}dxdy$$
   where D is the region above the line $x+y=2$ and inside the circle $x^2+y^2=4$

   (a) as interated integrals in cartesian coordinates;

   (b) as iterated integrals in polar coordinates

   then evaluate only once.

5. Evaluate $\iiint_B \frac{zdV}{\sqrt{x^2+y^2}}$ where $B = \{(x,y,z)\,; 1 \le z \le 4 \text{ and } z \ge x^2+y^2\}$.

6. Find the surface area of S

   where S is the part of $z = \sqrt{3x^2+3y^2}$ below the plane $x+z=4$.

7. Find $\oint_c \mathbf{F}\cdot\mathbf{ds}$

   where $\mathbf{F} = (y^3x + \cos(x^2), e^{y^2} + \sin(\pi x))$ and c is boundary of the triangle T from $(0,2)$ to $(2,2)$ to $(2,0)$ and back to $(0,2)$.

8. Show that for any smooth vector field $\mathbf{F}(x,y)$ and any smooth real-valued function $\phi(x,y)$

   $div\,(\phi\mathbf{F}) = grad\phi \bullet \mathbf{F} + \phi div\ \mathbf{F}$ $\quad \nabla\cdot(\phi\mathbf{F}) = \nabla\phi\cdot\mathbf{F} + \phi(\nabla\cdot\mathbf{F})$.

9. Evaluate $\oint_c \mathbf{F}\cdot\mathbf{ds}$ where $\mathbf{F} = (x^2+y, y^3-x, z^4)$

   and c is given as $\{x^2+y^2=4\} \cap \{2x-3y+z=2\}$ oriented positively.

   .

10. Find the flux of $\mathbf{F} = (x^2, y^2, z^2)$ outward
from the closed surface $S = \{x^2 + y^2 + 4(z-1)^2 = 4\}$.

11. Evaluate $\int_c \mathbf{F} \bullet \mathbf{ds}$ where $\mathbf{F} = (y, x, 2z)$ and
$c = \{z = 2xy\} \cap \{x^2 + y^2 = 2\}$ from $A\,(-1, 1, -2)$ to $B\,(1, 1, 2)$.

12. For the vector field $\mathbf{F}\,(x, y, z) = (\arctan z(x^2 + y^2)\,, \ln(1 + y^2 + z^2), y\ e^{xyz})$
find *div* $\mathbf{F}$ and *curl* $\mathbf{F}$ in the domain.