MATHEMATICS 353
QUIZ #1 R

R

WINTER 2006

1. Find ∂D - boundary of D. Is D closed? Open? Bounded? Sketch the set.

   (a) $D = (x, y) \mid \sqrt{x} < \sqrt{y}\}$
   (b) $D = \{(x, y, z) \mid x^2 + y^2 \leq z \leq 9\}$ [5]

SOLUTION

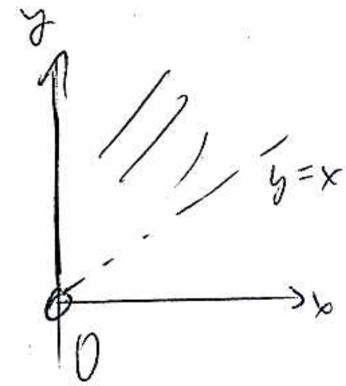
for a)

it must $x \geq 0$,and $y > 0$ then $0 \leq x < y$

in the first quadrant and above the line $y = x$;so the set UNBDD

and the boundary is $\partial D = \{y = x, x \geq 0\} \cup \{x = 0, y > 0\}$

first part is out,other parts are in,so **neither open nor closed.**



for b)

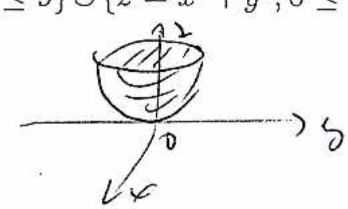
$z = x^2 + y^2$ is half of a paraboloid, $z \geq 0$, $z = 9$ is a horizontal plane

so D is the set of points on or above the paraboloid ,and on or below the plane $z = 9$

therefore the set BDD,and the boundary $\partial D = \{z = 9, x^2 + y^2 \leq 9\} \cup \{z = x^2 + y^2, 0 \leq z \leq 9\}$

"lid" and "cup"

both parts are included in D ,so the set is CLOSED



2. Find all local extrema of $f(x, y) = 4xy^2 - 2x^2 - y^2$ in its domain.Explain. [5]

SOLUTION

f is defined, continous,differentiable everywhere,for critical points solve

$f_x = 4y^2 - 4x = 0 \qquad x = y^2$

$f_y = 8xy - 2y = 2y\,(4x - 1) = 0 \qquad x = \frac{1}{4}$ or $y = 0$

we got 3 critical points $(0, 0), \left(\frac{1}{4}, \frac{1}{2}\right), \left(\frac{1}{4}, -\frac{1}{2}\right)$

for Second Derivative Test

$f_{xx} = -4 \qquad f_{xy} = 8y \qquad f_{yy} = 8x - 2$

Now A ...B....C....D

$(0, 0)$	-4	0	-2	*neg*	*loc.* max
$\left(\frac{1}{4}, \frac{1}{2}\right)$	-4	4	0	*pos*	*saddle*
$\left(\frac{1}{4}, -\frac{1}{2}\right)$	-4	-4	0	*pos*	*saddle*