The University of Calgary **Department of Mathematics and Statistics**
MATH 353-01 **Quiz #1T** **Winter 2006**

Name:______________________________**I.D.#:**__________________

1. Sketch the given set S.Find the boundary ∂S .Is the set S closed,open,bounded?

   (a) $S = \left\{ (x, y); \dfrac{3}{x} \le y \right\}$.

   (b) $S = \left\{ (x, y, z); \sqrt{x^2 + y^2} \le z \le 4 \right\}$. [5]

2. Find all local extrema of $f(x, y) = e^x \left(y^2 + 2xy\right)$ in the domain. [5]

Solutions **For 1a)**

$\dfrac{3}{x}$ is defined only $x \neq 0$ so y-axis is out ,

all point on or above the hyperbola $y = \dfrac{3}{x}$except the y-axis

therefore the set is UNBDD

we can see that the boundary $\partial S = \{x = 0\} \cup \{y = \dfrac{3}{x}\}$

the axis are out ,the hyperbola is in so the set S is **neither open nor closed..**

For 1b)

we can see that $z = 4$ is a horizon.plane ,$z = \sqrt{x^2 + y^2}$is a cone (half)

and the set is inside and on the cone (above the xy-plane) and below or on the plane $z = 4$

$\partial S = \left\{ (x, y, z); \sqrt{x^2 + y^2} = z, 0 \le z \le 4 \right\} \cup \{(x, y, z); x^2 + y^2 \le 2, z = 4\}$ "*cone*"+"*lid*"

Thus $\partial S \subset S$, the whole boundary is inside the set ,so S is **closed and bounded.**

For 2) the function f is defined and differentiable everywhere

for critical points solve

$f_x = e^x \left(y^2 + 2xy + 2y\right) = ye^x \left(y + 2x + 2\right) = 0$

$f_y = e^x \left(2y + 2x\right) = 0$ thus $y = -x$

if $y = -x$ from the first equ. $-xe^x \left(x + 2\right) = 0$ so $x = 0$ or $x = -2$

2 critical points $(0, 0), (-2, 2)$

$f_{xx} = ye^x \left(y + 2x + 4\right)$ $f_{xy} = e^x \left(2y + 2x + 2\right)$ $f_{yy} = 2e^x$

points	A	B	C	D
$(0, 0)$	0	2	2	4
$(2, -2)$	$4e^2$	$2e^2$	$2e^2$	$-4e^4$

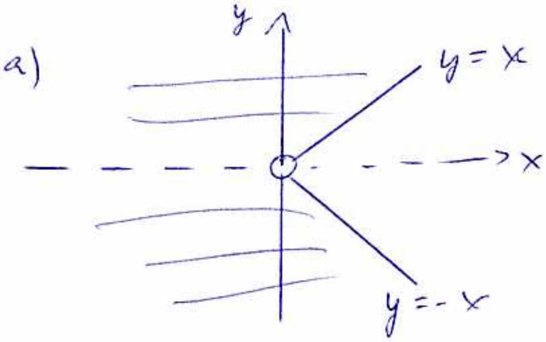
$(0, 0)$ is a **saddle point** since the discriminant $D = B^2 - AC > 0$

$(2, -2)$ is a **loc.min** since $A > 0, D < 0$

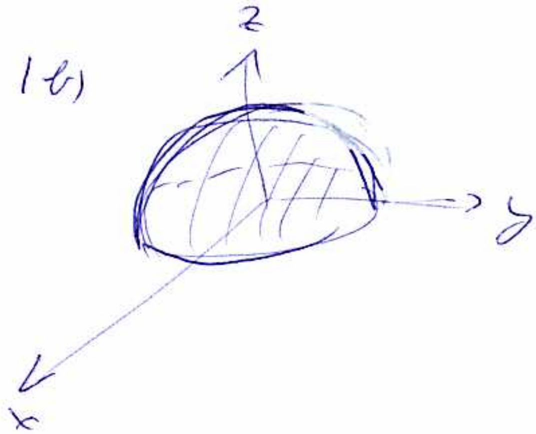
Quiz #1

T 04   Tue 10am

1a)

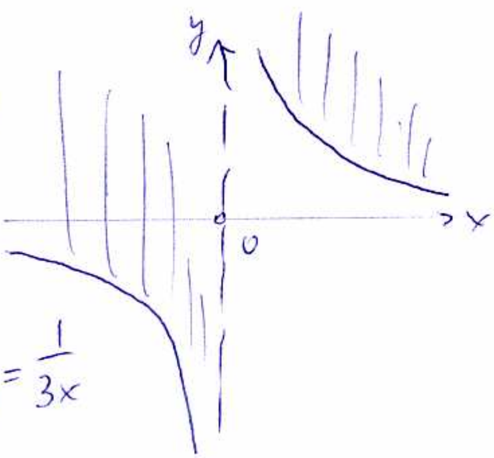


1b)

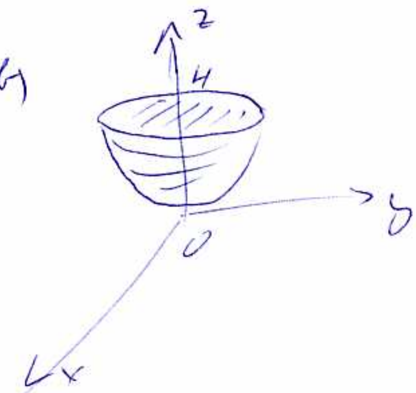


T 05   Tue 3pm

1a)

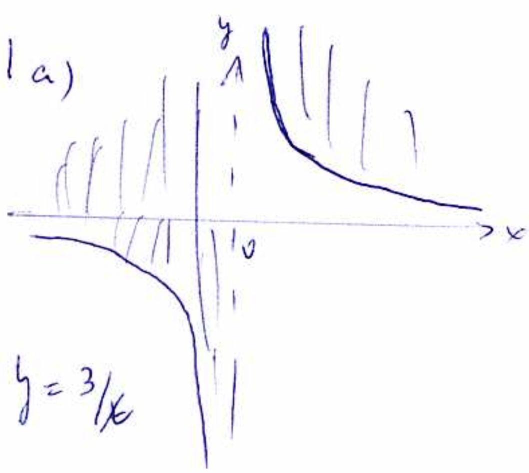


1b)



T 01 + 02   Tue 2pm

1a)



1b)