MATH 353-02 **Quiz #2T3pm** **Winter, 2006**

Name:____________________________**I.D.#:**_________________

1. Find abs.maximum and minimum values of $f(x,y) = xy^2$ on the set $S = \{(x,y); 2x^2 + y^2 \leq 6\}$. [5]

2. For $\iint_D \frac{y}{\sqrt{x}}\, dA$ where D is the region in the first quadrant between $y = \sqrt{x}$ and $x = 4$

   (a) sketch the region D;

   (b) set up BOTH iterated integrals and evaluate one of them. [5]

Solution

For 1)

first ,for critical points inside : solve $\nabla f = \overrightarrow{0}$

$f_x = y^2 = 0 \qquad f_y = 2xy = 0 \qquad (x,0)$ for any x

critical points on the boundary $\partial S = \{g(x,y) = 2x^2 + y^2 = 6\}$.

solve $\nabla f = \lambda \nabla g \qquad y^2 = \lambda 4x \qquad 2xy = \lambda 2y$

if $y = 0$ then $x = \pm\sqrt{3}$;if $y \neq 0$ then $x = \lambda$ and $y^2 = 4x^2$

back to the ellipse $6x^2 = 6 \qquad x = \pm 1, y = \pm 2$

check the values of f : $f(x,0) = 0, f(\pm\sqrt{3}, 0) = 0$

$f(1, \pm 2) = 4$ maxima;$f(-1, \pm 2) = -4$ minima

For 2)

$0 \leq x \leq 4 \qquad 0 \leq y \leq \sqrt{x}$ or $\qquad 0 \leq y \leq 2 \qquad y^2 \leq x \leq 4$

and

$$\iint_D \frac{y}{\sqrt{x}}\, dA = \int_0^4 \left(\frac{1}{\sqrt{x}} \int_0^{\sqrt{x}} y dy \right) dx = \int_0^4 \frac{1}{\sqrt{x}} \left[\tfrac{y^2}{2}\right]_{y=0}^{y=\sqrt{x}} dx = \tfrac{1}{2} \int_0^4 \sqrt{x} dx =$$

$$= \left[\frac{x^{\frac{3}{2}}}{3} \right]_0^2 = \frac{8}{3}$$

OR

$$\iint_D \frac{y}{\sqrt{x}}\, dA = \int_0^2 \left(y \int_{y^2}^4 \tfrac{1}{\sqrt{x}} dx \right) dy = \int_0^2 y \left[2\sqrt{x}\right]_{x=y^2}^{x=4} dy = 2 \int_0^2 y \left[2 - y\right] dy =$$

$$= 2\left[y^2 - \tfrac{y^3}{3}\right]_0^2 = \frac{8}{3}.$$