The University of Calgary
Department of Mathematics and Statistics
MATH 353-01 Quiz #3 2pm Winter 2006

Name:______________________________**I.D.#:**________________

1. u=Set up the integral $\displaystyle\iint_D \frac{y\sin(\frac{\pi}{2}(x^2+y^2))}{\sqrt{x^2+y^2}}dxdy$

   where $D = \{(x,y); y \geq x \geq 0, 1 \leq x^2+y^2 \leq 4\}$ as iterated integrals in both cartesian and polar coordinates and then evaluate. [6]

2. Evaluate the integral $\displaystyle\iint_D e^{-\frac{y}{x}}\ dA$ if it is convergent,

   where $D = \{(x,y), x > 0, x^2 \leq y \leq 2x^2\}$ .Sketch the set. [4]

Solution
For 1)
using polar coord. $\displaystyle I = \iint_D \frac{y\sin(\frac{\pi}{2}(x^2+y^2))}{\sqrt{x^2+y^2}}dxdy = \iint_{D^*} \frac{r\sin\theta\sin(\frac{\pi}{2}r^2)}{r}rdrd\theta =$
where $D^* = \{\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}, 1 \leq r \leq 2\}$ so
$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\theta d\theta \int_1^2 r\sin(\tfrac{\pi}{2}r^2)dr = [-\cos\theta]_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left[-\tfrac{1}{\pi}\cos(\tfrac{\pi}{2}r^2)\right]_1^2 = \frac{-1}{\pi\sqrt{2}}$$
in cartesian coord,sketch the set, we have to split ,it is easier with vert. slicing
intersection of $y = x$ and circles at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\sqrt{2}, \sqrt{2}\right)$
for $0 \leq x \leq \frac{1}{\sqrt{2}}$ $\quad \sqrt{1-x^2} \leq y \leq \sqrt{4-x^2}$
for $\frac{1}{\sqrt{2}} < x \leq \sqrt{2}$ $\quad x \leq y \leq \sqrt{4-x^2}$ so
$$I = \int_0^{\frac{1}{\sqrt{2}}} (\int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} \frac{y\sin(\frac{\pi}{2}(x^2+y^2))}{\sqrt{x^2+y^2}}dy)dx + \int_{\frac{1}{\sqrt{2}}}^{\sqrt{2}} (\int_x^{\sqrt{4-x^2}} \frac{y\sin(\frac{\pi}{2}(x^2+y^2))}{\sqrt{x^2+y^2}}dy)dx.$$
if horiz. slicing - 3 parts!
For 2)
the set is unbdd
$$\iint_D e^{-\frac{y}{x}}\ dA = \int_0^\infty (\int_{x^2}^{2x^2} e^{-\frac{y}{x}}dy)dx = \int_0^\infty \left[-xe^{-\frac{y}{x}}\right]_{y=x^2}^{y=2x^2} dx = \int_0^\infty \left[-xe^{-2x} + xe^{-x}\right] dx =$$
by parts
$$= \left[\tfrac{1}{2}xe^{-2x} - xe^{-x}\right]_0^\infty - \int_0^\infty \left[\tfrac{1}{2}e^{-2x} - e^{-x}\right] dx = 0 + \left[\tfrac{1}{4}e^{-2x} - e^{-x}\right]_0^\infty =$$
$$= \frac{3}{4} \text{ using } \lim_{x\to\infty} xe^{-ax} = \lim_{x\to\infty}\frac{x}{e^{ax}} = 0 \text{ by L'H.R.for any } a > 0.$$
.