The University of Calgary
Department of Mathematics and Statistics
MATH 353
Quiz #4T 3pm

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Name:______________________________**I.D.#:**__________________

1. For $\mathbf{F}(x,y) = (2xe^{x^2+y^2} + \dfrac{1}{x}, kye^{x^2+y^2} - y)$ find the value for k so that the field is conservative ,then find a potential. [3]

2. Evaluate $\int_c x^2\, ds$ and c is the intersection of $\{\ z = \ln x, y$ any$\}$ and the plane $1 = x + y$ between $A(1,0,0)$ and $B\,(2,-1,\ln 2)$. [4]

3. Find $\int_c \mathbf{F}\cdot d\mathbf{s}$ where $\mathbf{F}(x,y,z) = (xz, z, -y)$ is given by $\mathbf{r}\,(t) = (t, \sin(\pi t), \cos(\pi t))$ from $A(0,0,1)$ and $B\,(1,0,-1)$. [3]

SOLUTION

For 1)

$F_1 = 2xe^{x^2+y^2} + \dfrac{1}{x}$ and $F_2 = kye^{x^2+y^2} - y$ for $x \neq 0$

$(F_1)_y = \frac{\partial}{\partial y}\left(2xe^{x^2+y^2} + \dfrac{1}{x}\right) = 4xye^{x^2+y^2}$

$(F_2)_x = \frac{\partial}{\partial x}\left(kye^{x^2+y^2} - y\right) = 2kxye^{x^2+y^2}$ so $k = 2$

then

$$f_x = 2xe^{x^2+y^2} + \frac{1}{x} \text{ and } f_y = 2ye^{x^2+y^2} - y$$

$f = \int f_x dx = \int (2xe^{x^2+y^2} + \dfrac{1}{x})dx + c(y) = e^{x^2+y^2} + \ln|x| + c(y)$

$f_y = 2ye^{x^2+y^2} + 0 + c'(y) = 2ye^{x^2+y^2} - y \qquad c'(y) = -y$

and $c(y) = -\frac{y^2}{2}$ together $f(x,y) = e^{x^2+y^2} + \ln|x| - \frac{1}{2}y^2 + c$

For 2)

parametrization choose $x = t$ then $y = 1-t, z = \ln t$

so $\mathbf{r}\,(t) = (t, 1-t, \ln t)$ for $t \in [1,2]$

$\mathbf{r}'\,(t) = \left(1, -1, \frac{1}{t}\right)$ and $\|\mathbf{r}'(t)\| = \sqrt{2 + \frac{1}{t^2}} = \dfrac{\sqrt{2t^2+1}}{t}$

$\displaystyle\int_c x^2 ds = \int_1^2 t\sqrt{2t^2+1}dt = (\text{ subst.}u = 2t^2+1) = \frac{1}{4}\int_3^9 \sqrt{u}du =$

$= \frac{1}{6}\left[u^{\frac{3}{2}}\right]_3^9 = \frac{1}{6}\left[3^3 - 3\sqrt{3}\right] = \frac{1}{2}\left(9 - \sqrt{3}\right).$

For 3)

$\mathbf{r}(t) = (t, \sin(\pi t), \cos(\pi t))$ $t \in [0,1]$ $\qquad \mathbf{r}'(t) = (1, \pi\cos\pi t, -\pi\sin\pi t)$

then the field on $c : \mathbf{F} \circ \mathbf{r} = (t\cos\pi t, \cos\pi t, -\sin\pi t)$

$\int_c \mathbf{F} \cdot d\mathbf{s} = \int_0^1 \mathbf{F} \cdot \mathbf{r}' dt = \int_0^1 \left(t\cos\pi t + \pi\cos^2\pi t + \pi\sin^2\pi t\right) dt =$ (by parts)=

$= \left[\frac{1}{\pi} t \sin\pi t - \frac{1}{\pi}\int \sin\pi t dt\right]_0^1 + \pi = 0 + \dfrac{1}{\pi^2}\left[\cos\pi t\right]_0^1 + \pi =$

$= \pi - \dfrac{2}{\pi^2}.$