INTEGRATION RULES

$$\int (Af(x) + Bg(x))\,dx = A\int f(x)\,dx + B\int g(x)\,dx$$

$$\int f'(g(x))\,g'(x)\,dx = f(g(x)) + C$$

$$\int U(x)\,dV(x) = U(x)\,V(x) - \int V(x)\,dU(x)$$

$$\int_b^a f'(x)\,dx = f(b) - f(a)$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

ELEMENTARY INTEGRALS

$$\int x^r\,dx = \frac{1}{r+1}x^{r+1} + C \text{ if } r \neq -1$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \sin x\ dx = -\cos x + C$$

$$\int \cos x\ dx = \sin x + C$$

$$\int \sec^2 x\ dx = \tan x + C$$

$$\int \csc^2 x\ dx = -\cot x + C$$

$$\int \sec x \tan x\ dx = \sec x + C$$

$$\int \csc x \cot x\ dx = -\csc x + C$$

$$\int \tan x\ dx = \ln|\sec x| + C$$

$$\int \cot x\ dx = \ln|\sin x| + C$$

$$\int \sec x\ dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\ dx = \ln|\csc x - \cot x| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C \quad (a > 0,\ |x| < a)$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \quad (a > 0)$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C \quad (a > 0)$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C \quad (a > 0,\ |x| > a)$$

TRIGONOMETRIC INTEGRALS

$$\int \sin^2 x\ dx = \frac{x}{2} - \frac{1}{4}\sin 2x + C$$

$$\int \cos^2 x\ dx = \frac{x}{2} + \frac{1}{4}\sin 2x + C$$

$$\int \tan^2 x\ dx = \tan x - x + C$$

$$\int \cot^2 x\ dx = -\cot x - x + C$$

$$\int \sec^3 x\ dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$$

$$\int \csc^3 x\ dx = -\frac{1}{2}\csc x \cot x + \frac{1}{2}\ln|\csc x - \cot x| + C$$

$$\int \sin ax \sin bx\ dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C \text{ if } a^2 \neq b^2$$

$$\int \cos ax \cos bx\ dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C \text{ if } a^2 \neq b^2$$

$$\int \sin ax \cos bx\ dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + C \text{ if } a^2 \neq b^2$$

$$\int \sin^n x\ dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x\ dx$$

$$\int \cos^n x\ dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\ dx$$

$$\int \tan^n x\ dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\ dx \text{ if } n \neq 1$$

$$\int \cot^n x\ dx = \frac{-1}{n-1}\cot^{n-1} x - \int \cot^{n-2} x\ dx \text{ if } n \neq 1$$

$$\int \sec^n x\ dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\ dx \text{ if } n \neq 1$$

$$\int \csc^n x\ dx = \frac{-1}{n-1}\csc^{n-2} x \cot x + \frac{n-2}{n-1}\int \csc^{n-2} x\ dx \text{ if } n \neq 1$$

$$\int \sin^n x \cos^m x\ dx = -\frac{\sin^{n-1} x \cos^{m+1} x}{n+m} + \frac{n-1}{n+m}\int \sin^{n-2} x \cos^m x\ dx \text{ if } n \neq -m$$

$$\int \sin^n x \cos^m x\ dx = \frac{\sin^{n+1} x \cos^{m-1} x}{n+m} + \frac{m-1}{n+m}\int \sin^n x \cos^{m-2} x\ dx \text{ if } m \neq -n$$

$$\int x \sin x\ dx = \sin x - x\cos x + C$$

$$\int x \cos x\ dx = \cos x + x\sin x + C$$

$$\int x^n \sin x\ dx = -x^n \cos x + n\int x^{n-1}\cos x\ dx$$

$$\int x^n \cos x\ dx = x^n \sin x - n\int x^{n-1}\sin x\ dx$$

TRIGONOMETRIC IDENTITIES

$\sin^2 x + \cos^2 x = 1$

$\sec^2 x = 1 + \tan^2 x$

$\csc^2 x = 1 + \cot^2 x$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\sin(-x) = -\sin x$

$\sin(\pi - x) = \sin x$

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$\cos(-x) = \cos x$

$\cos(\pi - x) = -\cos x$

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

$\cos^2 x = \frac{1 + \cos 2}{2}$

QUADRATIC FORMULA

$Ax^2 + Bx + C = 0 \qquad x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

INTEGRALS INVOLVING $\sqrt{x^2 \pm a^2}$ $(a > 0)$

(If $\sqrt{x^2 - a^2}$, assume $|x| > a > 0$.)

$$\int \sqrt{x^2 \pm a^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

$$\int \frac{\sqrt{x^2 + a^2}}{x}\,dx = \sqrt{x^2 + a^2} - a\ln\left|\frac{a + \sqrt{x^2 + a^2}}{x}\right| + C$$

$$\int \frac{\sqrt{x^2 - a^2}}{x}\,dx = \sqrt{x^2 - a^2} - a\tan^{-1}\frac{\sqrt{x^2 - a^2}}{a} + C$$

$$\int x^2\sqrt{x^2 \pm a^2}\,dx = \frac{x}{8}\left(2x^2 \pm a^2\right)\sqrt{x^2 \pm a^2} - \frac{a^4}{8}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}}\,dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \mp \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

$$\int \frac{\sqrt{x^2 \pm a^2}}{x^2}\,dx = -\frac{\sqrt{x^2 \pm a^2}}{x} + \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

$$\int \frac{dx}{x^2\sqrt{x^2 \pm a^2}} = \mp\frac{\sqrt{x^2 \pm a^2}}{a^2 x} + C$$

$$\int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}} + C$$

$$\int \left(x^2 \pm a^2\right)^{3/2} dx = \frac{x}{8}\left(2x^2 \pm 5a^2\right)\sqrt{x^2 \pm a^2} + \frac{3a^4}{8}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

INTEGRALS INVOLVING $\sqrt{a^2 - x^2}$ $(a > 0,\ |x| < a)$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

$$\int \frac{\sqrt{a^2 - x^2}}{x}\,dx = \sqrt{a^2 - x^2} - a\ln\left|\frac{a + \sqrt{a^2 - x^2}}{x}\right| + C$$

$$\int \frac{x^2}{\sqrt{a^2 - x^2}}\,dx = -\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

$$\int x^2\sqrt{a^2 - x^2}\,dx = \frac{x}{8}\left(2x^2 - a^2\right)\sqrt{a^2 - x^2} + \frac{a^4}{8}\sin^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{x^2\sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C$$

$$\int \frac{\sqrt{a^2 - x^2}}{x^2}\,dx = -\frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - x^2}}{x}\right| + C$$

$$\int \frac{dx}{(a^2 - x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 - x^2}} + C$$

$$\int \left(a^2 - x^2\right)^{3/2} dx = \frac{x}{8}\left(5a^2 - 2x^2\right)\sqrt{a^2 - x^2} + \frac{3a^4}{8}\sin^{-1}\frac{x}{a} + C$$

INTEGRALS OF INVERSE TRIGONOMETRIC FUNCTIONS

$$\int \sin^{-1} x\ dx = x\sin^{-1} x + \sqrt{1 - x^2} + C$$

$$\int \tan^{-1} x\ dx = x\tan^{-1} x - \frac{1}{2}\ln\left(1 + x^2\right) + C$$

$$\int \sec^{-1} x\ dx = x\sec^{-1} x - \ln\left|x + \sqrt{x^2 - 1}\right| + C \qquad (x > 1)$$

$$\int x\sin^{-1} x\ dx = \frac{1}{4}\left(2x^2 - 1\right)\sin^{-1} x + \frac{x}{4}\sqrt{1 - x^2} + C$$

$$\int x\tan^{-1} x\ dx = \frac{1}{2}\left(x^2 + 1\right)\tan^{-1} x - \frac{x}{2} + C$$

EXPONENTIAL AND LOGARITHMIC INTEGRALS

$$\int xe^x dx = (x - 1)\,e^x + C$$

$$\int x^n e^x dx = x^n e^x - n\int x^{n-1}e^x dx$$

$$\int \ln x\ dx = x\ln x - x + C$$

$$\int x^n \ln x\ dx = \frac{x^{n+1}}{n+1}\ln x - \frac{x^{n+1}}{(n+1)^2} + C, \qquad (n \neq -1)$$

$$\int x^n (\ln x)^m\,dx = \frac{x^{n+1}}{n+1}(\ln x)^m \quad -\frac{m}{n+1}\int x^n (\ln x)^{m-1}\,dx \qquad (n \neq -1)$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2 + b^2}\left(a\sin bx - b\cos bx\right) + C$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2 + b^2}\left(a\cos bx + b\sin bx\right) + C$$

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