The University of Calgary   Department of Mathematics and Statistics
MATH 353-02   Quiz #1T   Winter 2006

Name:______________________________**I.D.#:**__________________

1. Sketch the given set S.Find the boundary ∂S .Is the set S closed,open,bounded?

   (a) $S = \left\{(x, y); \frac{1}{x} \leq 3y\right\}$.

   (b) $S = \{(x, y, z); x^2 + y^2 \leq z \leq 4\}$. [5]

2. Find all local extrema of $f(x, y) = e^y (x^2 - 2xy)$ in the domain. [5]

Solutions   For 1a)

$\frac{1}{x}$ is defined only $x \neq 0$ so y-axis is out ,

all point on or above the hyperbola $y = \frac{1}{3x}$except the y-axis

therefore the set is UNBDD

we can see that the boundary $\partial S = \{x = 0\} \cup \{y = \frac{1}{3x}\}$

the axis are out ,the hyperbola is in so the set S is **neither open nor closed..**

For 1b)

we can see that $z = 4$ is a horizon.plane ,$z = x^2 + y^2$ is a paraboloid

and the set is inside and on the paraboloid above the xy-plane and below and on the plane $z = 4$

$\partial S = \{(x, y, z); x^2 + y^2 = z, 0 \leq z \leq 4\} \cup \{(x, y, z); x^2 + y^2 \leq 4, z = 4\}$ "*cup*" + "*lid*"

Thus $\partial S \subset S$, the whole boundary is inside the set ,so S is **closed and bounded.**

For 2)   the function f is defined and differentiable everywhere

for critical points solve

$f_x = e^y (2x - 2y) = 0 \ldots\ldots x = y$

$f_y = e^y (x^2 - 2xy - 2x) = xe^y (x - 2y - 2) = 0$

if $x = y$ from the second equ. $xe^y (-x - 2) = 0$ so $x = 0$ or $x = -2$

2 critical points $(0, 0), (-2, -2)$

$f_{xx} = 2e^y$ $f_{xy} = e^y (2x - 2y - 2)$ $f_{yy} = xe^y (x - 2y - 4)$

points	A	B	C	D
$(0, 0)$	2	-2	0	4
$(-2, -2)$	$2e^{-2}$	$-2e^{-2}$	$4e^{-2}$	$-4e^{-4}$

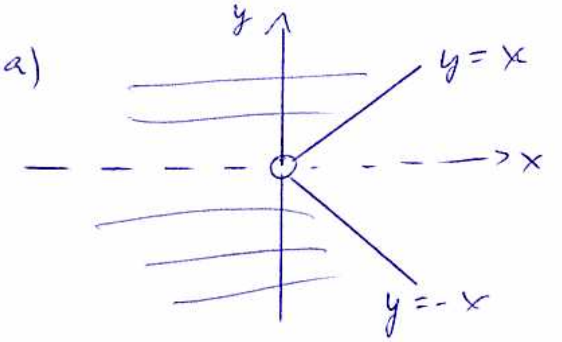
$(0, 0)$ is a **saddle points** since the discriminant $D = B^2 - AC > 0$

$(-2, -2)$ is a **loc. min** since $A > 0, D < 0$

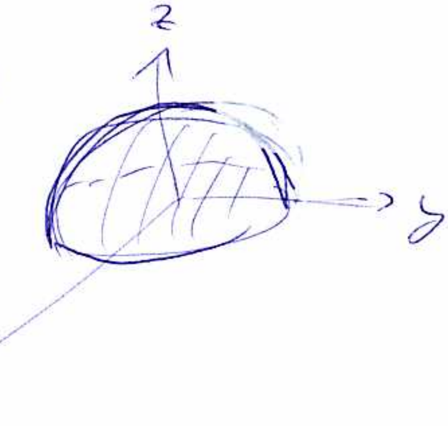
Quiz #1

T 04 Tue 10am

1 a)

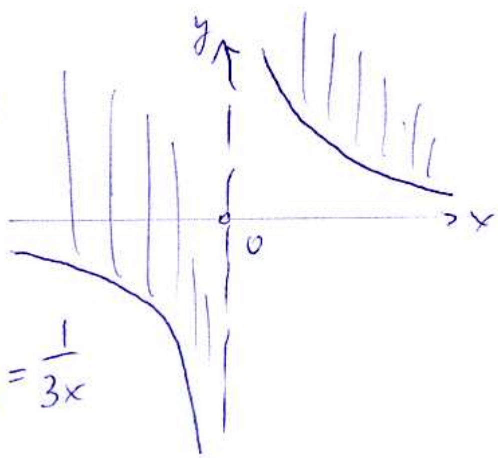


1 b)

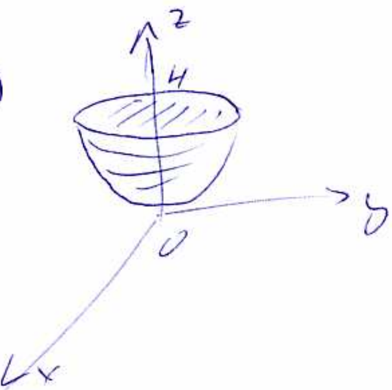


T 05 Tue 3pm

1 a)

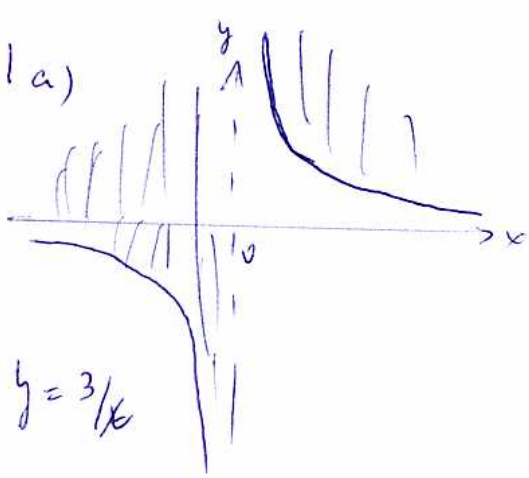


1 b)



T 01 + 02 Tue 2pm

1 a)



1 b)