The University of Calgary
Department of Mathematics and Statistics
MATH 353-01
Quiz #2R

Winter, 2006

Name:______________________________**I.D.#:**____________________

1. Find max/min values of $f(x,y) = x^4 + y^4$ on $2x^2 + y^2 = 20$. [3]
2. Find max/min values of $f(x,y) = x^4 + y^4$ on the triangle with verices $(0,0), (1,0)$ and $(0,1)$. [3]
3. Evaluate $\iint_D x^3 \cos(xy) dA$ where D is the region in the first quadrant below the parabola $y = x^2$ and the line $x = 1$. [4]

SOLUTION

For 1)

Since f is continous and the ellipse is closed and bounded we have at least one max and one min

to find the points solve

$\nabla f = \lambda \nabla g$ with $g(x,y) = 2x^2 + y^2 = 20$

$4x^3 = \lambda 4x$

$4y^3 = \lambda 2y$ if $xy \neq 0$ then $\lambda = x^2 = 2y^2$ back to the ellipse

$4y^2 + y^2 = 5y^2 = 20 \to y^2 = 4$ $y = \pm 2, x = \pm 2\sqrt{2}$

all combination $\left(, \pm 2 \pm 2\sqrt{2}\right), \left(, \pm 2 \mp 2\sqrt{2}\right)$

all have the same value as $f\left(2, 2\sqrt{2}\right) = 16 + 64 = 80$

we have to consider $xy = 0$ $x = 0$ or $y = 0$

$\left(0, \pm 2\sqrt{5}\right)$ $f = 400$ $\left(\pm\sqrt{10}, 0\right)$ $f = 100$

maximum value is 400,minimum velue is 80.

For 2)

C.P inside $\nabla f = \mathbf{0}$ $x = y = 0$

C.P.on the boundary- 3 parts

$B_1 = \{y = 0, x \in [0,1]\}$ $f|_{B_1} = x^4$ only ends

$B_2 = \{x = 0, y \in [0,1]\}$ $f|_{B_2} = y^4$ only ends

$B_3 = \{x + y = 1, x \in [0,1]\}$ $f|_{B_3} = x^4 + (1-x)^4 = h(x)$

$h'(x) = 4x^3 - 4(1-x)^3 = 0$ if $x^3 = (1-x)^3$ $x = 1 - x$ $x = y = \frac{1}{2}$

all C.P. $(0,0), (1,0)$, $(0,1), \left(\frac{1}{2}, \frac{1}{2}\right)$ now velues of f

$f(0,0) = 0 \qquad$ minimum

$f(1,0) = f\ (0,1) = 1 \qquad$ maximum, $f\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2^4} + \frac{1}{2^4} = \dfrac{1}{8}$

For 3)

$D = \{0 \le x \le 1\ , 0 \le y \le x^2\}$

$$\iint_D x^3 \cos(xy) dA = \int_0^1 x^3 \left(\int_0^{x^2} \cos(xy) dy \right) dx = \int_0^1 x^3 \left[\frac{\sin(xy)}{x} \right]_{y=0}^{y=x^2} dx =$$

$$= \int_0^1 (x^2 \sin(x^3)\, dx = (\text{ by subst.} u = x^3, du = 3x^2 dx) = \tfrac{1}{3} \int_0^1 (\sin(u)\, du$$

$$= \tfrac{1}{3} \left[-\cos u \right]_0^1 = \frac{1 - \cos 1}{3}.$$