MATH 353-02 **Quiz #2T** **Winter, 2006**

Name:____________________________**I.D.#:**_________________

1. Find abs.maximum and minimum values of $f(x,y) = x^3 + y^3$ on the set $S = \{(x,y); x^2 + 2y^2 \le 9\}$. [5]

2. For $\iint_D \sqrt{2-y^2}\, dA$ where D is the region in the first quadrant below the line $y = x$ and inside the circle $x^2 + y^2 = 2$

   (a) sketch the region D;

   (b) set up BOTH iterated integrals and evaluate one of them. [5]

Solution

For 1)

first ,for critical points inside : solve $\nabla f = \overrightarrow{0}$

$f_x = 3x^2 = 0 \qquad f_y = 3y^2 = 0$

so $(0,0)$ is critical point

critical points on the boundary $\partial S = \{g(x,y) = x^2 + 2y^2 = 9\}$.

solve $\nabla f = \lambda \nabla g \qquad 3x^2 = \lambda 2x \qquad 3y^2 = \lambda 4y$

from the first equation $x = 0$ or $\lambda = \dfrac{3x}{2}$

from the second equation $y = 0$ or $\lambda = \dfrac{3}{4}y$

if $x = 0$ from the set $y = \pm\frac{3}{\sqrt{2}}$ so C.P.s $\left(0, \pm\frac{3}{\sqrt{2}}\right)$

if $y = 0$ from the set $x = \pm 3$ so C.P.s $(\pm 3, 0)$

if $xy \neq 0 \qquad \lambda = \dfrac{3x}{2} = \dfrac{3y}{4} \qquad y = 2x$ then from the set

$9x^2 = 9 \qquad x = \pm 1$ and $y = \pm 2$ 4 more C.P.

now values of f : $f(0,0) = 0 \qquad f\left(0, \pm\frac{3}{\sqrt{2}}\right) = \pm\frac{27}{2\sqrt{2}}$

$f(\pm 1, \pm 2) = \pm 9 \qquad f(\pm 3, 0) = \pm 27$.....max/min

For 2)

for the region we need the intersection of $y = x$ and $x^2 + y^2 = 2$

with $x \ge 0$ and $y \ge 0 \qquad (1,1)$

it is easier to slice it horizontally:line is the left end, circle the right one

$0 \le y \le 1 \qquad y \le x \le \sqrt{2-y^2}$

so

$$\iint_D \sqrt{2-y^2}\, dA = \int_0^1 \left(\sqrt{2-y^2} \int_y^{\sqrt{2-y^2}} dx \right) dy = \int_0^1 \left(\sqrt{2-y^2} \left[\sqrt{2-y^2} - y \right] \right) dy =$$

$$= \int_0^1 \left[2 - y^2 - y\sqrt{2-y^2} \right] dy = \left[2y - \tfrac{1}{3}y^3 \right]_0^1 + \tfrac{1}{2}\int_2^1 \sqrt{u}du = \frac{5}{3} - \tfrac{1}{3}\left[u^{\frac{3}{2}} \right]_1^2 = \tfrac{1}{3}\left(6 - 2\sqrt{2}\right)$$

(subst. $u = 2 - y^2, du = -2ydxy$)

to slice it vertically we have to split since we have two tops:line + circle for $0 \le x \le 1 \qquad 0 \le y \le x$

for $1 \le x \le \sqrt{2} \qquad 0 \le y \le \sqrt{2-x^2} \qquad$ thus

$$\iint_D \sqrt{2-y^2}\ dA = \int_0^1 \left(\int_0^x \sqrt{2-y^2} dy \right) dx + \int_1^{\sqrt{2}} \left(\int_0^{\sqrt{2-x^2}} \sqrt{2-y^2} dy \right) dx$$