MATH 353-01 **Quiz #2T2pm** **Winter, 2006**

Name:____________________________**I.D.#:**________________

1. Find abs.maximum and minimum values of $f(x,y) = x^2y$ on the set $S = \{(x,y); x^2 + 2y^2 \leq 6\}$. [6]

2. For $\iint_D \frac{x}{\sqrt{y}}\, dA$ where D is the region in the first quadrant between $y = x^2$ and $y = 4$

   (a) sketch the region D;

   (b) set up BOTH iterated integrals and evaluate one of them.. [4]

Solution

For 1)

first ,for critical points inside : solve $\nabla f = \overrightarrow{0}$

$f_x = 2xy = 0 \quad f_y = x^2 = 0 \quad (0, y)$ for any y

critical points on the boundary $\partial S = \{g(x,y) = x^2 + 2y^2 = 6\}$.

solve $\nabla f = \lambda \nabla g \quad 2xy = \lambda 2x \quad x^2 = \lambda 4y$

if $x = 0$ then $y = \pm\sqrt{3}$;if $x \neq 0$ then $y = \lambda$ and $x^2 = 4y^2$

back to the ellipse $6y^2 = 6 \quad y = \pm 1, x = \pm 2$

check the values of f : $f(0,y) = 0, f(0, \pm\sqrt{3}) = 0$

$f(\pm 2, 1) = 4$ maxima;$f(\pm 2, -1) = -4$ minima

For 2)

intersection : $y = 4 = x^2 \quad x = 2$

so $\quad 0 \leq x \leq 2 \quad x^2 \leq y \leq 4$ or $\quad 0 \leq y \leq 4 \quad 0 \leq x \leq \sqrt{y}$

and

$$\iint_D \frac{x}{\sqrt{y}} dA = \int_0^2 \left(x \int_{x^2}^4 \frac{1}{\sqrt{y}} dy \right) dx = \int_0^2 \left(x \left[2\sqrt{y} \right]_{y=x^2}^{y=4} \right) dx = 2 \int_0^2 x \left[2 - x \right] dx =$$

$$= 2 \int_0^2 (2x - x^2)\, dx = 2 \left[x^2 - \frac{x^3}{3} \right]_0^2 = 2 \left[4 - \tfrac{8}{3} \right] = \frac{8}{3}$$

OR

$$\int_0^4 \left(\frac{1}{\sqrt{y}} \int_0^{\sqrt{y}} x dx \right) dy = \int_0^4 \frac{1}{\sqrt{y}} \left[\frac{x^2}{2} \right]_{x=0}^{x=\sqrt{y}} dy = \frac{1}{2} \int_0^4 \sqrt{y} dy =$$

$$= \frac{1}{3} \left[y^{\frac{3}{2}} \right]_0^4 = \frac{8}{3}.$$