The University of Calgary
Department of Mathematics and Statistics
MATH 353-01
Quiz #3R

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Name:____________________________**I.D.#:**__________________

1. Set up the integral $\iint\limits_D \frac{x}{\sqrt{x^2+y^2}}dxdy$ where $D = \{(x,y)\,;x \geq 1, y \geq 0, x^2+y^2 \leq 2\}$ as iterated integrals in both cartesian coordinates, and polar coordinates, and then evaluate (only once). [6]

2. Evaluate the integral $\iint\limits_D e^{-x^2y}\, dA$ if it is convergent, where $D = \{(x,y); 2 \leq x; 0 \leq y \leq \frac{1}{x^3}\}$.Sketch the set. [4]

Solution
For 1)
sketch the set,intersection of the line and circle is at $(1,1)$

so $$\iint\limits_D \frac{x}{\sqrt{x^2+y^2}}dxdy = \int_0^1 \left(\int_1^{\sqrt{2-y^2}} \frac{x}{\sqrt{x^2+y^2}}dx\right)dy = \int_0^1 \left(\left[\sqrt{x^2+y^2}\right]_{x=1}^{x=\sqrt{2-y^2}}\right)dy =$$
$$= \int_0^1 \left(\sqrt{2}-\sqrt{1+y^2}\right)dy \;=\; \text{(Table)}=\; \sqrt{2} - \left[\tfrac{y}{2}\sqrt{1+y^2}+\tfrac{1}{2}\ln(y+\sqrt{1+y^2}\right]_0^1 \;=\; \tfrac{1}{2}(\sqrt{2}+\ln(1+\sqrt{2})$$

OR
$$\iint\limits_D \frac{x}{\sqrt{x^2+y^2}}dxdy = \int_1^{\sqrt{2}} x\left(\int_0^{\sqrt{2-x^2}} \frac{dy}{\sqrt{x^2+y^2}}\right)dx = \text{harder}$$

in polar coord. $$\iint\limits_D \frac{x}{\sqrt{x^2+y^2}}dxdy = \iint\limits_{D^*} \frac{r\cos\theta}{r} r dr d\theta$$

where $D^* = \{0 < r \leq \sqrt{2}\ ,\ r \geq \frac{1}{\cos\theta}, \theta \in \left[0, \frac{\pi}{4}\right]\}$
$$= \int_0^{\frac{\pi}{4}} \cos\theta\Big(\int_{\frac{1}{\cos\theta}}^{\sqrt{2}} r\ dr\Big)d\theta = \tfrac{1}{2}\int_0^{\frac{\pi}{4}} \cos\theta\big(2-\tfrac{1}{\cos^2\theta}\big)d\theta = \int_0^{\frac{\pi}{4}} \cos\theta d\theta - \frac{1}{2}\int_0^{\frac{\pi}{4}} \sec\theta d\theta =$$
$$= \text{(Table)}\,[\sin\theta]_0^{\frac{\pi}{4}} - [\ln(\sec\theta+\tan\theta]_0^{\frac{\pi}{4}} = \tfrac{1}{\sqrt{2}} - \tfrac{1}{2}\ln\left(\sqrt{2}+1\right).$$

For 2)
the function is bounded,but the set is unbdd

$$\iint\limits_D e^{-x^2y}\ dA = \int\limits_2^{\infty} (\int\limits_0^{\frac{1}{x^3}} e^{-x^2y}dy)dx = \int\limits_2^{\infty} \tfrac{-1}{x^2}\left[e^{-x^2y}\right]_{y=0}^{y=\frac{1}{x^3}} dx = \int\limits_2^{\infty} \tfrac{-1}{x^2}\left[e^{-\frac{1}{x}} - 1\right] dx =$$

$$= \int\limits_2^{\infty} \frac{1}{x^2}dx - \int\limits_2^{\infty} \tfrac{1}{x^2}e^{-\frac{1}{x}}dx = \left[\frac{-1}{x}\right]_2^{\infty} - \int\limits_{-\frac{1}{2}}^{0} e^u du \text{ (by subst.} u = \tfrac{-1}{x}) = 0 + \tfrac{1}{2} - 1 + e^{\frac{-1}{2}} = \tfrac{1}{\sqrt{e}} - \tfrac{1}{2}.$$