The University of Calgary
Department of Mathematics and Statistics
MATH 353-02
Quiz #3T10am

Winter,2006

Name:______________________________**I.D.#:**__________________

1. Set up the integral $\iint\limits_D ye^{\sqrt{x^2+y^2}}dxdy$

   where $D = \{(x, y); y \geq x \geq 0, x^2 + y^2 \leq 4\}$ as iterated integrals in both cartesian and polar coordinates and then evaluate. [6]

2. Is the integral $\iint\limits_D \frac{1}{(x^2+y)^2}dxdy$,where $D = \{(x, y); x \geq 2, 0 \leq y \leq x^2\}$

   convergent or divergent? If ,yes, evaluate.Sketch the set.

[4]

Solution
For 1)
in polar coord.:$I = \iint\limits_D ye^{\sqrt{x^2+y^2}}dxdy = \iint\limits_{D^*} r\sin\theta e^r r dr d\theta$
where $D^* = \{\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}, 0 \leq r \leq 2\}$ so
$I = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\theta d\theta \int\limits_0^2 r^2 e^r dr = [-\cos\theta]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[e^2 e^r - \int 2re^r dr\right]_0^2$ (by parts twice) =
$= \frac{1}{\sqrt{2}}\left[r^2e^r - 2re^r + 2e^r\right]_0^2 = \frac{1}{\sqrt{2}}\left[2e^2 - 2\right] = \sqrt{2}\left(e^2 - 1\right);$
in cartesian coord.: intersection of $y = x$ and the circle is at $\left(\sqrt{2}, \sqrt{2}\right)$
so $0 \leq x \leq \sqrt{2}$ $\quad x \leq y \leq \sqrt{4-x^2}$ and
$I = \int\limits_0^{\sqrt{2}}\left(\int\limits_x^{\sqrt{4-x^2}} ye^{\sqrt{x^2+y^2}}dy\right)dx.$
.
For 2)
the set is unbounded, function bounded
$\iint\limits_D \frac{1}{(x^2+y)^2}dxdy = \int\limits_2^{\infty}\left(\int\limits_0^{x^2} \frac{1}{(x^2+y)^2}dy\right)dx = \int\limits_2^{\infty}\left[\frac{-1}{x^2+y}\right]_{y=0}^{y=x^2} dx =$
0
$= \int\limits_2^{\infty}\left[\frac{-1}{2x^2} + \frac{1}{x^2}\right]dx = \frac{1}{2}\int\limits_2^{\infty}\frac{1}{x^2}dx = \frac{1}{2}\left[-\frac{1}{x}\right]_2^{\infty} = \frac{1}{4}$
since $\lim\limits_{x\to\infty} \frac{-1}{x} = 0$