The University of Calgary
Department of Mathematics and Statistics
MATH 353-02 Quiz #3T 3pm Winter 2006

Name:______________________________**I.D.#:**__________________

1. Set up the integral $\iint\limits_D \frac{x\cos(\frac{\pi}{2}(x^2+y^2))}{\sqrt{x^2+y^2}}dxdy$

   where $D = \{(x,y); x \geq y \geq 0, 1 \leq x^2+y^2 \leq 4\}$ as iterated integrals in both cartesian and polar coordinates and then evaluate. [6]

2. Evaluate the integral $\iint\limits_D ye^{-\frac{y^2}{x}}\ dA$ if it is convergent,

   where $D = \{(x,y), x > 0, x \leq y \leq 2x\}$ .Sketch the set. [4]

Solution
For 1)

using polar coord. $I = \iint\limits_D \frac{x\cos(\frac{\pi}{2}(x^2+y^2))}{\sqrt{x^2+y^2}}dxdy = \iint\limits_{D^*} \frac{r\cos\theta\cos(\frac{\pi}{2}r^2)}{r}rdrd\theta =$

where $D^* = \{0 \leq \theta \leq \frac{\pi}{4}, 1 \leq r \leq 2\}$ so

$$I = \int_0^{\frac{\pi}{4}} \cos\theta d\theta \int_1^2 r\cos(\tfrac{\pi}{2}r^2)dr = [\sin\theta]_0^{\frac{\pi}{4}} \left[\tfrac{1}{\pi}\sin(\tfrac{\pi}{2}r^2)\right]_1^2 = \frac{-1}{\pi\sqrt{2}}$$

in cartesian coord,sketch the set, we have to split ,it is easier with horiz. slicing
intersection of $y = x$ and circles at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\sqrt{2}, \sqrt{2}\right)$
for $0 \leq y \leq \frac{1}{\sqrt{2}}$ $\quad \sqrt{1-y^2} \leq x \leq \sqrt{4-y^2}$
for $\frac{1}{\sqrt{2}} < y \leq \sqrt{2}$ $\quad y \leq x \leq \sqrt{4-y^2}$ so

$$I = \int_0^{\frac{1}{\sqrt{2}}} \left(\int_{\sqrt{1-y^2}}^{\sqrt{4-y^2}} \frac{x\cos(\frac{\pi}{2}(x^2+y^2))}{\sqrt{x^2+y^2}}dx\right)dy + \int_{\frac{1}{\sqrt{2}}}^{\sqrt{2}} \left(\int_y^{\sqrt{4-y^2}} \frac{x\cos(\frac{\pi}{2}(x^2+y^2))}{\sqrt{x^2+y^2}}dx\right)dy.$$

if verical slicing - 3 parts!

For 2)

the set is unbdd

$$\iint\limits_D ye^{-\frac{y^2}{x}}\ dA = \int_0^\infty \left(\int_x^{2x} ye^{-\frac{y^2}{x}}dy\right)dx = \int_0^\infty \left[-\tfrac{1}{2}xe^{-\frac{y^2}{x}}\right]_{y=x}^{y=2x} dx = \tfrac{-1}{2}\int_0^\infty \left[xe^{-4x} - xe^{-x}\right]dx =$$

by parts

$$= \tfrac{1}{2}\int_0^\infty xe^{-x}dx - \tfrac{1}{2}\int_0^\infty xe^{-4x}dx = \left[-\tfrac{1}{2}xe^{-x}\right]_0^\infty + \int_0^\infty \tfrac{1}{2}e^{-x}dx + \left[\tfrac{1}{8}xe^{-4x}\right]_0^\infty - \int_0^\infty \tfrac{1}{8}e^{-4x}dx =$$

using $\lim\limits_{x\to\infty} xe^{-ax} = \lim\limits_{x\to\infty} \frac{x}{e^{ax}} = 0$ by L'H.R.for any $a > 0$

$$= 0 + \left[-\tfrac{1}{2}e^{-x}\right]_0^\infty - 0 + \left[\tfrac{1}{32}e^{-4x}\right]_0^\infty = \tfrac{1}{2} - \tfrac{1}{32} = \frac{15}{32}.$$

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