# THE UNIVERSITY OF CALGARY
# DEPARTMENT OF MATHEMATICS & STATISTICS
# MATHEMATICS 353 QUIZ #4R
# Winter 2006

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1. Find the value for $k$ so that $\mathbf{F}(x,y,z) = \left( \frac{kx}{\sqrt{x^2+y^2}} + \cos x, \frac{y}{\sqrt{x^2+y^2}} - e^{-y} \right)$ is conservative ,and then find a potential.$f$. [3]

2. Find $\int_c \mathbf{F} \cdot \mathbf{ds}$ where $\mathbf{F}(x,y,z) = (-3y, 0, x)$ and $\mathbf{c}$ is the intersection of the plane $2x - 3y + z = 1$ and the cylinder $x^2 + y^2 = 4$, oriented counterclockwise. [4]

3. Evaluate $\int_c f \, ds$ where $f(x,y,z) = yz$ and the curve $c$ is given by $\mathbf{r}(t) = (\ln t, 3t, 4t)$ between $A(0,1,2)$ and $B(\ln 2, 2, 4)$. [3]

**Solutions.**

**For 1)** for any $(x,y) \neq (0,0)$

$$(F_1)_y = \frac{\partial}{\partial y}\left(\frac{kx}{\sqrt{x^2+y^2}}\right) = \frac{-2kxy}{2(x^2+y^2)^{\frac{3}{2}}}$$

$$(F_2)_x = \frac{\partial}{\partial x}\left(\frac{y}{\sqrt{x^2+y^2}}\right) = \frac{-y2x}{2(x^2+y^2)^{\frac{3}{2}}} \qquad \text{so } k = 1$$

then $\quad f_x = \frac{kx}{\sqrt{x^2+y^2}} + \cos x \qquad f_y = \frac{y}{\sqrt{x^2+y^2}} - e^{-y}$

$$f = \int \frac{x}{\sqrt{x^2+y^2}} + \cos x \; dx + c(y) = \sqrt{x^2+y^2} + \sin x + c(y)$$

$$f_y = \frac{2y}{2\sqrt{x^2+y^2}} + c'(y) = \frac{y}{\sqrt{x^2+y^2}} - e^{-y} \text{ thus } c' = -e^y$$

and

$$f(x,y,z) = \sqrt{x^2+y^2} + \sin x + e^{-y} + c$$

**For 2)**

Since the field is not conservative we have to parametrize the curve e.g

( $z = 1 - 2x + 3y$)

$\mathbf{c} : \mathbf{r}(t) = (2\cos t, 2\sin t, 1 - 4\cos t + 6\sin t), t \in [0, 2\pi]$

tangent vector

$\mathbf{r}'(t) = (-2\sin t, 2\cos t, 4\sin t + 6\cos t)$ and

the field evaluated on the curve $\mathbf{F}(\mathbf{r}(t))$=$(-6\sin t, 0, 2\cos t)$

So

$$\int_c \mathbf{F}\cdot\mathbf{ds} = \int_0^{2\pi} \mathbf{F}(t)\cdot\mathbf{r}'(t)dt = \int_0^{2\pi} \left(12\sin^2 t + 8\cos t\sin t + 12\cos^2 t\right) dt =$$

$$= 12\cdot 2\pi + \left[\tfrac{8}{2}\sin^2 t\right]_0^{2\pi} = 24\pi$$

**For 3)**

for $t > 0$ $\qquad \mathbf{r}'(t) = \left(\frac{1}{t}, 3, 4\right) \qquad \|\mathbf{r}'(t)\| = \sqrt{\frac{1}{t^2}+5} = \dfrac{\sqrt{1+25t^2}}{t}$

$f(\mathbf{r}(t) = 12t^2$ and $t = 1$ for $A, t = 2$ for $B$ thus

$$\int_c f\ ds = \int_1^2 12t^2\frac{\sqrt{1+25t^2}}{t}dy = 12\int_1^2 t\sqrt{1+25t^2}dt\,(u = 1+25t^2) = \tfrac{6}{25}\int_{26}^{101}\sqrt{u}du =$$

$$= \tfrac{6}{25}\left[\tfrac{2}{3}u^{\frac{3}{2}}\right]_{26}^{101} = \tfrac{4}{25}\left[101^{\frac{3}{2}} - 26^{\frac{3}{2}}\right].$$