The University of Calgary
Department of Mathematics and Statistics
MATH 353
Quiz #4T10am

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Name:____________________________**I.D.#:**__________________

1. For $\mathbf{F}(x,y) = (\frac{y}{1+x^2y^2} + 2x, \frac{kx}{1+x^2y^2} + y^2)$ find the value for k so that the field is conservative ,then find a potential. [3]

2. Evaluate $\int_c \sqrt{5+z^2}\, ds$ and c is given by $\mathbf{r}(t) = (t, t^2, 2t)$ between the.origin and $P(-1,1,-2)$. [3]

3. Find $\int_c \mathbf{F} \cdot d\mathbf{s}$ where $\mathbf{F}(x,y,z) = (z, 2y, -y)$ and c is the intersection of the plane $z = 2x$ and the paraboloid $z = x^2 + y^2$ oriented counterclockwise. [4]

SOLUTION

For 1)

$F_1 = \frac{y}{1+x^2y^2} + 2x$,and $F_2 = \frac{kx}{1+x^2y^2} + y^2$

for any point

$$(F_1)_y = \frac{\partial}{\partial y}\left(\frac{y}{1+x^2y^2} + 2x\right) = \frac{1+x^2y^2 - y2x^2y}{(1+x^2y^2)^2} = \frac{1-x^2y^2}{(1+x^2y^2)^2}$$

$$(F_2)_x = \frac{\partial}{\partial x}\left(\frac{kx}{1+x^2y^2} + y^2\right) = k\frac{1-x^2y^2}{(1+x^2y^2)^2} \text{ so } k=1 \text{ then } f=?$$

$f_x = \frac{y}{1+x^2y^2} + 2x$,and $f_y = \frac{kx}{1+x^2y^2} + y^2$

$$f = \int f_x dx = \int \left(\frac{y}{1+x^2y^2} + 2x\right) dx + c(y) = \arctan(xy) + x^2 + c(y)$$

$$f_y = \frac{x}{1+x^2y^2} + 0 + c'(y) = F_2 = \frac{x}{1+x^2y^2} + y^2$$

$c'(y) = y^2$and $\qquad c(y) = \frac{1}{3}y^3$

together $f(x,y) = \arctan(xy) + x^2 + \frac{1}{3}y^3 + c$

For 2)

$\mathbf{r}(t) = (t, t^2, 2t)\,, t \in [-1, 0]$ $\mathbf{r}'(t) = (1, 2t, 2)$

$\|\mathbf{r}'(t)\| = \sqrt{5+4t^2}$ and f on $c = \sqrt{5+4t^2}$

$$\int_c \sqrt{5+z^2} ds = \int_{-1}^{0} f(r(t))\, \|\mathbf{r}'(t)\|\, dt = \int_{-1}^{0} (5+4t^2)\, dt = \left[5t + \tfrac{4}{3}t^3\right]_{-1}^{0} = 5 + \frac{4}{3}. = \frac{19}{3}.$$

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For 3)

intersection of $z = 2x$ and $z = x^2 + y^2$ $\qquad 2x = x^2 + y^2$

$1 = (x-1)^2 + y^2$ $\qquad x = 1 + \cos t, y = \sin t$ and $z = 2x = 2 + 2\cos t$

$\mathbf{r}(t) = (1 + \cos t, \sin t, 2 + 2\cos t), t \in [0, 2\pi]$

$\mathbf{r}'(t) = (-\sin t, \cos t, -2\sin t)$

then the field on $c : \mathbf{F} \circ \mathbf{r} = (2 + 2\cos t, 2\sin t, -\sin t)$

$\int_c \mathbf{F} \cdot d\mathbf{s} = \int_0^{2\pi} \mathbf{F} \cdot \mathbf{r}' dt = \int_0^{2\pi} \left(-2\sin t - 2\sin t\cos t + 2\sin t\cos t + 2\sin^2 t\right) dt =$

$= [2\cos t]_0^{2\pi} + \int_0^{2\pi} (1 + \cos 2t)\, dt = 2\pi.$