The University of Calgary
Department of Mathematics and Statistics
MATH 353
Quiz #4T 2pm

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Name:______________________________**I.D.#:**__________________

1. For $\mathbf{F}(x,y) = (\dfrac{2x}{x^2+y^2}+2, \dfrac{ky}{x^2+y^2}-\dfrac{1}{y})$ find the value for k so that the field is conservative ,then find a potential. [3]

2. Evaluate $\int_c z\ ds$ and c is the intersection of $\{\ z = e^y, x \text{ any}\}$ and the plane $1 = x + y$ between $A(0,1,e)$ and $B\,(1,0,1)$. [4]

3. Find $\int_c \mathbf{F}\cdot d\mathbf{s}$ where $\mathbf{F}(x,y,z) = (z,y,2)$ is given by $\mathbf{r}\,(t) = (t^2, t, \cos(\pi t))$ from $A(0,0,1)$ and $B\,(1,1,-1)$. [3]

SOLUTION

For 1)

$F_1 = \dfrac{2x}{x^2+y^2}+2$ and $F_2 = \dfrac{ky}{x^2+y^2}-\dfrac{1}{y}$ for $y \neq 0$

$(F_1)_y = \frac{\partial}{\partial y}\left(\dfrac{2x}{x^2+y^2}+2\right) = -\dfrac{4xy}{(x^2+y^2)^2}$

$(F_2)_x = \frac{\partial}{\partial x}\left(\dfrac{ky}{x^2+y^2}-\dfrac{1}{y}\right) = -\dfrac{2kxy}{(x^2+y^2)^2}$ so $k = 2$

then

$$f_x = \frac{2x}{x^2+y^2}+2 \text{ and } f_y = \frac{2y}{x^2+y^2}-\frac{1}{y}$$

$f = \int f_x dx = \int (\dfrac{2x}{x^2+y^2}+2)dx + c(y) = \ln(x^2+y^2)+2x+c(y)$

$f_y = \dfrac{2y}{x^2+y^2}+0+c'(y) = \dfrac{2y}{x^2+y^2}-\dfrac{1}{y} \qquad c'(y) = -\dfrac{1}{y}$ for $y \neq 0$

and $c(y) = -\ln|y|$ together $f(x,y) = \ln(x^2+y^2)+2x-\ln|y|+c$

For 2)

parametrization choose $y = t$ then $x = 1-t, z = e^t$

so $\mathbf{r}\,(t) = (1-t, t, e^t)$ for $t \in [0,1]$

$\mathbf{r}'\,(t) = (-1,1,e^t)$ and $\|\mathbf{r}'(t)\| = \sqrt{2+e^{2t}}$

$\int_c z\ ds = \int_0^1 e^t\sqrt{2+(e^t)^2}dt = (\text{ subst.}u = e^t) = \int_1^e \sqrt{2+u^2}du =$

$\left(a=\sqrt{2}\text{Table}\right)$

$=\frac{1}{2}\left[u\sqrt{2+u^2}+\ln(u+\sqrt{2+u^2})\right]_1^e=\frac{1}{2}\left[e\sqrt{2+e^2}+\ln(e+\sqrt{2+e^2})-\sqrt{3}-\ln\left(1+\sqrt{3}\right)\right]$

$=.$

For 3)

$\mathbf{r}\left(t\right)=\left(t^2,t,\cos\pi t\right),t\in[0,1] \qquad \mathbf{r}'\left(t\right)=\left(2t,1,-\pi\sin\pi t\right)$

then the field on $c:\mathbf{F}\circ\mathbf{r}=(\cos\pi t,t,2)$

$\int_c \mathbf{F}\cdot d\mathbf{s}=\int_0^1 \mathbf{F}\cdot\mathbf{r}'dt=\int_0^1 \left(2t\cos\pi t+t-2\pi\sin\pi t\right)dt=$ (by parts)=

$=\left[\frac{2}{\pi}t\sin\pi t-\frac{2}{\pi}\int\sin\pi t dt\right]_0^1+\frac{1}{2}+2\left[\cos\pi t\right]_0^1=0+\frac{2}{\pi^2}\left[\cos\pi t\right]_0^1+\frac{1}{2}-4=$

$=\frac{-4}{\pi^2}-\frac{7}{2}.$