The University of Calgary
Department of Mathematics and Statistics
MATH 353
Quiz #5R

Winter 2006

Name:______________________________**I.D.#:**__________________

1. Evaluate $\iint\limits_S \dfrac{dS}{\sqrt{1+4x^2}}$ and S is the part of the vertical surface $y = 1 + x^2$ between two planes $z = 0$ and $6x - 3y - z = -3$. [5]

2. Find $\iint\limits_S \mathbf{F} \bullet \mathbf{dS}$ where $\mathbf{F}(x, y, z) = (xz, yz, z)$ and S is the part of the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 2$ oriented outward. [5]

SOLUTION

For 1)

$(0 \le z \le 6x - 3y + 3)$

S is vertical so parametrization

$\mathbf{r}(u, v) : x = u \qquad y = 1 + u^2 \qquad z = v$

$\qquad u \in [??] \qquad 0 \le v \le 6u - 3 - 3u^2 + 3 = 3u(2 - u)$

so.necessary $3u(2 - u) \ge 0 \qquad 0 \le u \le 2.....R$

$\frac{\partial}{\partial u}\mathbf{r}(u, v) = (1, 2u, 0) \qquad \frac{\partial}{\partial v}\mathbf{r}(u, v) = (0, 0, 1)$

$\mathbf{n} = \left(\frac{\partial}{\partial u}\mathbf{r} \times \frac{\partial}{\partial v}\mathbf{r}\right) = (2u, -1, 0) \qquad \|\mathbf{n}\| = \sqrt{4u^2 + 1}$

$$\iint\limits_S \frac{dS}{\sqrt{1+4x^2}} = \iint\limits_R \frac{1}{\sqrt{1+4u^2}} \|\mathbf{n}\| \, dudv = \int_0^2 \tfrac{1}{\sqrt{4u^2+1}} \sqrt{4u^2+1} \left(\int_0^{6u-3u^2} dv \right) du =$$

$$= \int_0^2 (6u - 3u^2) \, du = \left[3u^2 - u^3\right]_0^2 = 4$$

For 2)

S is given by $z = \sqrt{x^2 + y^2}$ for $(x, y) \in D = \{x^2 + y^2 \le 4\}$

$\mathbf{n} = \left(\dfrac{x}{\sqrt{x^2+y^2}}, \dfrac{y}{\sqrt{x^2+y^2}}, -1 \right)$ ...since outward means downward

$\mathbf{F}$ on $S \qquad \mathbf{F}(x, y, z) = (x\sqrt{x^2+y^2}, y\sqrt{x^2+y^2}, \sqrt{x^2+y^2})$

and $\mathbf{F} \bullet \mathbf{n} = x^2 + y^2 - \sqrt{x^2+y^2}$ then

$$\iint\limits_S \mathbf{F} \bullet \mathbf{dS} = \iint\limits_D \mathbf{F} \bullet \mathbf{n} \, dxdy = \iint\limits_D (x^2 + y^2 - \sqrt{x^2+y^2}) dxdy = \int_0^{2\pi} \int_0^2 (r^3 - r^2) dr d\theta =$$

$$= 2\pi \left[\tfrac{r^4}{4} - \tfrac{r^3}{3} \right]_0^2 = 2\pi \left[4 - \tfrac{8}{3} \right] = \frac{8}{3}\pi.$$