The University of Calgary
Department of Mathematics and Statistics
MATH 353
Quiz #5T10am

Winter 2006

Name:______________________________**I.D.#:**__________________

1. Evaluate $\iint\limits_S x^2 \; dS$ and S is the part of the cylinder $\qquad 4 = x^2 + y^2$

   between two planes $z = 0$ and $x + y + z = 3$. [5]

2. Find $\iint\limits_S \mathbf{F} \bullet \mathbf{dS}$ where $\mathbf{F}(x, y, z) = (y, xy, z)$ and S is the part of

   the paraboloid $z = x^2 + y^2$ inside the cylinder $x^2 + y^2 = 4$ oriented inward. [5]

SOLUTION

For 1)

S is vertical so parametrization

$\mathbf{r}(u, v) : x = 2\cos u \qquad y = 2\sin u \qquad z = v$

$\qquad u \in [0, 2\pi] \qquad 0 \le v \le 3 - 2\cos u. - 2\sin u ...R$

$\frac{\partial}{\partial u}\mathbf{r}(u, v) = (-2\sin u, 2\cos u, 0) \qquad \frac{\partial}{\partial v}\mathbf{r}(u, v) = (0, 0, 1)$

$\mathbf{n} = \left(\frac{\partial}{\partial u}\mathbf{r} \times \frac{\partial}{\partial v}\mathbf{r}\right) = (2\cos u, 2\sin u, 0) \qquad \|\mathbf{n}\| = 2$

$$\iint\limits_S x^2 \; dS = 8\iint\limits_R \cos^2 u \; du dv = 8\int_0^{2\pi} \cos^2 u \left(\int_0^{3-2\cos u - 2\sin u} dv \right) du =$$

$$= 8\int_0^{2\pi} \cos^2 u \,(3 - 2\cos u - 2\sin u)\, du = 8\int_0^{2\pi} 3\cos^2 u - 2\cos^3 u - 2\cos^2 u \sin u du =$$

$$= 12\int_0^{2\pi} (1 + \cos 2\theta)\, d\theta - 0 = 24\pi \qquad \text{since } 2\pi\text{−periodicity.}$$

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For 2)

S is given by $z = x^2 + y^2$ for $(x, y) \in D = \{x^2 + y^2 \le 4\}$

$\mathbf{n} = (-2x, -2y, 1)$ ...since inward means upward

$\mathbf{F}$ on $S \qquad \mathbf{F}(x, y, z) = (y, xy, x^2 + y^2)$

and $\mathbf{F} \bullet \mathbf{n} = -2xy - 2xy^2 + x^2 + y^2$ then

$$\iint\limits_S \mathbf{F} \bullet \mathbf{dS} = \iint\limits_D \mathbf{F} \bullet \mathbf{n} \; dxdy = \iint\limits_D (-2xy - 2xy^2 + x^2 + y^2) dxdy = 0 + \int_0^{2\pi}\int_0^{2} (r^3) dr d\theta =$$

since 2x is odd and the set is symetrical in$\pm x$

$= 2\pi \left[\frac{r^4}{4}\right]_0^2 = 8\pi.$