The University of Calgary
Department of Mathematics and Statistics
MATH 353
Quiz #5T 3pm

Winter 2006

Name:______________________________**I.D.#:**_________________

1. Evaluate $\iint\limits_S z\, x^2 dS$ and S is the part of the sphere $x^2+y^2+z^2=2$,between two planes $z=0$ and $z=1$. [5]

2. Find $\iint\limits_S \mathbf{F} \bullet \mathbf{dS}$ where $\mathbf{F}(x,y,z)=(yz,x,xyz)$ and S is the part of the plane $x+y=1$ in the first octant below the plane $x+y+z=3$ oriented in the direction of positive y. [5]

SOLUTION

For 1)

S is given by $z=\sqrt{2-x^2-y^2}$ for $(x,y)\in D=\{1\le x^2+y^2\le\sqrt{2}\}$

since $\quad 0\le z\le 1$

$$\mathbf{n}=\left(\frac{x}{\sqrt{2-x^2-y^2}},\frac{y}{\sqrt{2-x^2-y^2}},1\right) \qquad \|\mathbf{n}\|=\sqrt{\tfrac{x^2+y^2}{2-x^2-y^2}+1}=\frac{\sqrt{2}}{\sqrt{2-x^2-y^2}}$$

$$\iint\limits_S zx^2\ dS=\iint\limits_D x^2\sqrt{2-x^2-y^2}\frac{\sqrt{2}}{\sqrt{2-x^2-y^2}}\ dxdy\ (\text{ polar})=\sqrt{2}\int_0^{2\pi}\int_1^{\sqrt{2}} r^3\cos^2\theta\ drd\theta=$$

$$=\sqrt{2}\int_1^{\sqrt{2}} r^3\ dr\int_0^{2\pi}\frac{1+\cos 2\theta}{2}d\theta=\sqrt{2}\left[\tfrac{r^4}{4}\right]_1^{\sqrt{2}}\pi=\frac{3\sqrt{2}}{4}\pi.$$

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For 2)

S is vertical so parametrization $\quad \mathbf{r}(u,v)$:

$x=u \qquad y=1-u \qquad z=v$

$x\ge 0, y\ge 0 \qquad u\in[0,1] \qquad 0\le v\le 3-1=2....R$

$\frac{\partial}{\partial u}\mathbf{r}(u,v)=(1,-1,0) \qquad \frac{\partial}{\partial v}\mathbf{r}(u,v)=(0,0,1)$

$\mathbf{n}=\pm\left(\frac{\partial}{\partial u}\mathbf{r}\times\frac{\partial}{\partial v}\mathbf{r}\right)=(-1,-1,0)$ or$(1,1,0)$ ...y pos

$\mathbf{F}$ on $S \qquad \mathbf{F}=((1-u)v,u,....)$and $\mathbf{F}\bullet\mathbf{n}=$$v-uv++u$ then

$$\iint\limits_S \mathbf{F}\bullet\mathbf{dS}=\iint\limits_R \mathbf{F}\bullet\mathbf{n}\ dudv=$$

$$= \int_0^1 (\int_0^2 (v - uv + u) dv) du = \int_0^2 v dv - \int_0^1 u du \left[\frac{v^2}{2}\right]_0^2 + 2\left[\frac{u^2}{2}\right]_0^1 = 2 - 1 + 1 = 2.$$